

University of Ruhuna
Bachelor of Science General Degree
Level II (Semester II) Examination - June 2022

Subject: Mathematics

Course Unit: MAT221 β (Number Theory)

Time :Two (02) Hours

Answer All Questions.

1. a) Define the followings;
(i) A proper divisor.
(ii) A common divisor. [10 marks]
- b) Let a, b and c be any non zero integers. Show that;
(i) $a|b$ and $a|c \rightarrow a|(bm + cn)$ for any integers m, n .
(ii) $a|b$ and $a|c \rightarrow a^2|bc$. [20 marks]
- c) Evaluate $[5756, 246]$. [20 marks]
- d) Let r be the remainder when 750, 870 and 1050 are divided by d where $d > 1$. Find the value of d/r . [20 marks]
- e) Find the largest positive integer p such that $p^3 + 65$ is divisible by $p + 5$. [15 marks]
- f) Show that $m^2 + 17$ is divisible by 18 for infinitely many m , where $m \in \mathbb{Z}$. [15 marks]
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2. a) (i) Define a prime number.
(ii) Show that if $p|ab$, p being a prime, then $p|a$ or $p|b$.
(iii) If p is a prime and $k \geq 0$, show that, $\sigma(p^k) = (p^{k+1} - 1)/(p - 1)$.
Hence compute $\sigma(72)$.

[50 marks]

b) Transporting gas cylinders is typically done by trucks such that they are placed upright with the valve at the top. Suppose a company has to transport 8440 gas cylinders. Company belongs two types of trucks, one that transports 280 cylinders and one that transports 340 cylinders. Suppose the company transports full trucks, and the trucks return empty. Taking x as the number of small trucks and y as the number of large trucks,

(i) write down a linear diophantine equation for transporting gas cylinders using small and large trucks,

(ii) find all the possible ways to transport gas cylinders.

[50 marks]

3. a) Find the integer solutions for $84x \equiv 49 \pmod{119}$.

[15 marks]

b) Show that 41 divides $2^{20} - 1$.

[15 marks]

c) State the Chinese Remainder Theorem.

[10 marks]

d) Using Chinese Remainder Theorem, find the least positive integer solution x which leaves a remainder of 3, 2, 4 and 2 when divided by 5, 3, 7 and 4 respectively.

[60 marks]

4. a) Solve the quadratic congruence $2x^2 + 3x + 1 \equiv 0 \pmod{11}$.

[50 marks]

b) Find the value of the Legendre symbol $(18/43)$.

(Hint: You may use $(2/p) = (-1)^{(p^2-1)/8}$; where p is a prime integer).

[15 marks]

c) Explain what is the order of an integer modulo n .

[10 marks]

d) (i) Define a primitive root of the integer n .

(ii) Show that 2 is a primitive root of 5.

[15 marks]

e) Does the equation $x^4 + y^4 = z^4$ has solutions in the positive integers? Explain the reason for your answer.

[10 marks]
