



University of Ruhuna
Bachelor of Science General Degree
Level II (Semester II) Examination - June 2022

Subject: Mathematics
Course Unit: MAT224δ (Geometry)

Time: One (01) Hour

Answer ALL Questions.

1. a) (i) If a line makes angles α, β, γ with positive directions of the coordinate axes, show that, $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.
- (ii) Find the equation of the plane which passes through the point $(-1, 3, 2)$ and perpendicular to the two planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$.
- b) (i) A variable plane which is at a constant distance $3p$ from the origin O cuts the coordinate axes at the points A, B , and C . Show that the locus of the centroid of the triangle ABC is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$$

and that of the tetrahedron $OABC$ is

$$\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \right) = \frac{16}{9p^2}$$

- c) Find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

$$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

2. a) (i) A sphere passes through the origin and meets the axes in the points A, B, C . If the radius of the sphere is $4r$, show that the locus of the centroid of ABC triangle is the sphere with centre at the origin and radius $8r/3$.
- (ii) Find the equations of the tangent planes of the sphere

$$x^2 + y^2 + z^2 - 4x - 4y - 4z + 10 = 0$$

which are parallel to the plane $x - z = 0$.

- b) (i) Find the equations of the tangent planes to the conicoid

$$\frac{x^2}{4} + y^2 - 2z^2 = 1,$$

which are perpendicular to the line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-4}.$$

- (ii) The normal drawn at any point P on the hyperboloid of 2 sheets

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

cuts the ZOY plane at Q . Find the locus of the point R , which is on the line PQ , and so that $PR : RQ = 1 : 2$.
