University of Ruhuna

Bachelor of Science General Degree
Level III (Semester II) Examination - January 2022

Subject: Mathematics
Course Unit: MAT324β (Mathematical Models in Ecology)

Time :Two (02) Hours

Answer <u>All</u> Questions.

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1. a) Explain the followings;
   (i) Discrete dynamical system.
   (ii) Affine dynamical system.

[10 marks]

b) Red blood cells are the most common type of blood cell in human body. They are also known as RBCs, haematids or erythrocytes. Suppose that a person should normally have around 2.5×10^{13} red blood cells in the body at any moment and bone marrow produces about 1720×10^{8} per day. Assume that a fixed number of cells are produced every day and that a fixed proportion of existing cells die each day.
   (i) Formulate a mathematical model for predicting the population of red blood cells.
   (ii) Considering the equilibrium situation, find the percentage of red blood cells that die each day.
   Hence, find the percentage of cells normally surviving each day.

[20 marks]

c) Vitamin E is a fat-soluble vitamin with several benefits for our body. It is primarily stored in our plasma and liver. Suppose that 45% of the vitamin E in the plasma is filtered out by the kidneys each day and that 35% of the vitamin E in the plasma is absorbed into the liver each day. Also, assume that 5% of the vitamin E in the liver is absorbed back in to plasma each day. Take the daily intake of Vitamin E as $15mg$ and assume that it goes directly in to the plasma. Let $p(n)$ and $l(n)$ be the amount of vitamin E in plasma and liver at the beginning of the day n, respectively.
   (i) Draw a flow diagram to model the situation described above.
   (ii) Develop an affine dynamical system to model the situation depicted above.
   (iii) Find the values of $p(3)$ and $l(3)$, if $p(0)$ and $l(0)$ are given as 10 and 70 respectively.

[70 marks]

2. a) Consider the following dynamical system of two equations;

$$U_{(n)} = 0.2U_{(n-1)} - 0.7V_{(n-1)} + \lambda$$

$$V_{(n)} = 0.8U_{(n-1)} + 0.4V_{(n-1)} + \mu$$

If the equilibrium value of the system is (0, 20), find the values of λ and μ.

[10 marks]

b) Consider the species of butterflies which have a life cycle that starts with eggs laid in leaves or stems of a plant, which turn into caterpillars. Fully grown caterpillars then turn into pupa or chrysalis and finally, become adult butterflies. Only adults can reproduce. About 60% of the caterpillars survives to become pupae. About 75% of pupae survive to become adult butterflies. 5% of adult butterflies die each year. Suppose that on the average, around 42 caterpillars born each year for every 100 adult butterflies at the beginning of the year. Let $c(n)$ be the number of caterpillars, $p(n)$ be the number of pupae and $a(n)$ be the number of adult butterflies, at the beginning of the n^{th} year, just after caterpillars have been born.

(i) Using a flow diagram, develop a set of equations to represent the dynamical system described above.

(ii) Complete the following table;

n	c(n)	p(n)	a(n)
0	50	100	150
1	63	30	217.5
2	91.35	37.8	229.125
3	96.2325	54.81	246.018
4	103.3279	57.7395	274.8253
5	115.4266	61.99673	304.3886
6	– – –	– – –	– – –
7	– – –	– – –	– – –
8	– – –	– – –	– – –

(iii) If the equilibrium point of the system is (0, 0, 0), show that the population growth rate of pupae is about 10% each year.

[90 marks]

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3. a) Consider the dynamical system, $u(n) = -2u(n-1) + 3$

(i) Find the equilibrium point of the system.

(ii) Taking $u(0) = 4$, determine the stability of the equilibrium point, using graphical method (no need to use the graph sheet).
Use at least 7 points to plot the above graph.

[35 marks]

Continued.

b) Consider the dynamical system $u(n) = 0.1u^2(n-1) + 0.5u(n-1)$.

(i) Determine the equilibrium values for this dynamical system.

(ii) Using a cob web diagram, comment on the stability of each equilibrium value (use the graph sheet provided).

(iii) Find the maximum interval of stability for the stable equilibrium value.

[65 marks]

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4. a) Define the following terms;

(i) Carrying capacity

(ii) Intrinsic growth rate

(iii) A sustainable yield

(iv) Compensation model

[20 marks]

b) Let carrying capacity be L and the intrinsic growth rate be b in usual notation. Assume that the growth rate decreases linearly as population size increases.

(i) If the growth rate function r is given by $r = mu + b$, then show that $r = b - (b/L)u$.

(ii) Obtain the logistic equation for a population in usual notation.

(iii) If $b = 0.25$ and $L = 320$, find the equilibrium points of the logistic equation. Also, determine the stability of them using the calculus technique.

[45 marks]

c) Consider a population of fish where the growth rate function is given by;
$g = 0.3u - 0.00005u^2$
Take the harvest per year as 8%. Also, assume that the harvest is given by the equation $h = pu$ in usual notation.

(i) Find the positive equilibrium population size and sketch a graph using the functions given above (no need to use a graph sheet).

(ii) Does the species safe, if the harverst changes to 40% ? Explain the reason for your answer.

[35 marks]
