UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: January 2022

Module Number: IS6303 **Module Name: Mathematical Modelling**

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1.

a) i.) Briefly explain the stages of 'Mathematical Modelling Process'.

ii.) Name two problems that might be modelled mathematically. Why do you think mathematics may provide a key to solving each problem? What is the added value in each case?

[3 Marks]

b) i.) Suppose an object is brought into a room having a constant room temperature T_R. If the temperature of the object at time t is denoted by T, explain why

$$\frac{dT}{dt} = k(T - T_R)$$

is the differential equation that expresses Newton's law of cooling.

ii.) Suppose an object having a temperature of 30°C brought into a room where the room temperature is 22°C. The object takes 12 *min* to cool to 28°C. Using the mathematical model in part (*i*), calculate the time required to cool the object to 25°C.

[3 Marks]

c) Fluids A, B, C and D contain the substances S_1, S_2 and S_3 (concentrations in grams per liter) according to the table shown below.

	A	B	C	D
S_1	2.5	8.0	6.5	10.5
S_2	3.5	16.5	13.5	0.5
S_3	1.2	0.8	2.2	3.5

What is the concentration of S_3 in a mixture of these fluids that contains 80% (percent by volume) of fluids A and B and which contains $4gl^{-1}$ and $5gl^{-1}$ of the substances S_1 and S_2, respectively?

[6 Marks]

Q2.

a) i.) Briefly explain the importance of 'Dimensional Analysis' in mathematical modelling of real world problems.

ii.) Clearly state the 'Buckingham Pi Theorem'.

[2 Marks]

b) The sound power (W), from a turbulent jet through a nozzle is believed to depend on the jet centerline velocity (V), nozzle diameter (D), speed of sound (C) and fluid density (ρ). The temperature and composition of the fluid in the jet is the same as the ambient fluid.

i.) Using the Buckingham Pi theorem, find a relation between the sound power and the other parameter(s) of the system.

ii.) How would the sound power vary if the nozzle diameter is increased by three times whilst all other factors remaining the same?

[4 Marks]

c) A patient in a hospital is required to have at least 90 units of drug A and at most 120 units of drug B each day. Each gram of substance M contains 10 units of drug A and 5 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. Now suppose that both M and N contain an undesirable drug C (which needs to be minimized), 3 units per gram in M and 2 units per gram in N.

i.) State the objective function and the problem constraints.

ii.) Using the graphical method, find the amount of substances M and N (in grams) that should be mixed to meet the stated daily requirements whilst minimizing the intake of drug C. How many units of the undesirable drug C will be in this mixture?

[6 Marks]

Q3.

a) Briefly explain about 'Balanced and Unbalanced Transportation Problems' and the use of 'Dummy demand point and Dummy supply point' in transportation problems.

[2 Marks]

b) Briefly explain the procedures of 'Vogel's Approximation Model (*VAM*)' and 'Stepping-Stone Solution Method' used to solve transportation problems.

[2 Marks]

c) Oranges are grown, picked, and then stored in warehouses in Bibile, Badulla and Monaragala. These warehouses supply oranges to markets in Ampara, Ratnapura, Kandy and Galle. The following table shows the supply costs per truckload (Rs.1000s), supply and demand.

From	To				Supply
	Ampara	Ratnapura	Kandy	Galle	
Bibile	9	14	12	17	200
Badulla	11	10	6	8	200
Monaragala	12	8	15	7	100
Demand	125	175	100	150	

i.) Set up the transportation tableau for this problem and determine the initial solution using the Vogel's Approximation Model.

ii.) Find the total cost for the initial feasible solution.

iii.) Solve the problem using the Stepping-Stone Solution Method.

iv.) Are there multiple optimal solutions? If so, explain and identify them.

[8 Marks]

Q4.

a) Briefly explain what is meant by,

i.) 'Row reductions' and 'Column reductions' performed in assignment problems.

ii.) Prohibited Assignment.

[2 Marks]

b) A local Airline company must staff the daily flights between New York and Chicago as shown in the table below. Each of the Airline's crew lives in either New York or Chicago. Each day a crew must fly one New York-Chicago and one Chicago-New York flight with at least 1-hour downtime between flights. The Airline wants to schedule the crews to minimize the total downtime. Setup an assignment problem that can be used to accomplish this goal. (*Hint:* Let $x_{ij} = 1$ if the crew that flies flight i also flies flight j, and $x_{ij} = 0$ otherwise. If $x_{ij} = 1$, then a cost C_{ij} is incurred, corresponding to the downtime associated with a crew flying flight i and flight j). According to the flight schedule, it is noticeable that some assignments are not feasible.

Flight	Leave Chicago	Arrive New York	Flight	Leave New York	Arrive Chicago
1	6 a.m.	10 a.m.	1	7 a.m.	9 a.m.
2	9 a.m.	1 p.m.	2	8 a.m.	10 a.m.
3	12 noon	4 p.m.	3	10 a.m.	12 noon
4	3 p.m.	7 p.m.	4	12 noon	2 p.m.
5	5 p.m.	9 p.m.	5	2 p.m.	4 p.m.
6	7 p.m.	11 p.m.	6	4 p.m.	6 p.m.
7	8 p.m.	12 midnight	7	6 p.m.	8 p.m.

Find the flight assignments that minimize the total downtime using 'Hungarian' method. Calculate the total minimum downtime. Assume that at the end of the day, each crew must be in its home city.

[10 Marks]

Q5.

a) Find the equilibrium point, if it exists, for the following dynamical system. State and prove whether the equilibrium point is stable or unstable.

$$u(n) = 3u(n-1) + v(n-1) + 1$$

$$v(n) = 4u(n-1) - v(n-1) + 6$$

[2 Marks]

b) Vitamin A is stored primarily in human being's Plasma and Liver. Suppose that 25% of the Vitamin A in the Plasma is filtered out by the Kidneys each day and that 50% of the Vitamin A in the Plasma is absorbed into the Liver each day. Also assume that 15% of the Vitamin A in the Liver is filtered out and that 5% of the Vitamin A in the Liver is absorbed back into the Plasma each day. Suppose you have a daily intake of 2mg of Vitamin A, which goes directly into the Plasma.

(Assume that the 1st dose is taken at 1st day night and all conversions of Vitamin A happen during daytime from 2nd day onwards)

i.) Determine a discrete dynamical system for $P(t)$ and $L(t)$, the number of milligrams of Vitamin A in the Plasma and the Liver respectively, after t days of your 1st dose of Vitamin A.

ii.) Find the equilibrium amounts of Vitamin A in the Plasma and Liver.

[5 Marks]

c) The number of rabbits on an island is currently 800, and it is increasing by 12.5% per year. Let R(t) represent the number of rabbits on this island t years from now.

i.) Write down a discrete dynamical system along with an initial value for R(t).

ii.) Find an explicit formula for R(t).

iii.) How many years will it take for the rabbit population on this island to double in size?

[5 Marks]