UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: October 2022

Module Number: ME1202 Module Name: Introduction to Mechanical Engineering

[Three Hours]

[Answer **all questions**, each question carries **12** marks]

Clearly state all the assumptions that you may make.
To get full marks, make sure that you have answered with correct SI units and standard notations.
Take gravitational acceleration, **g = 9.8 m/s^2**

Q1. a) The acceleration (a) of a particle in a rectilinear motion is given by a function $f(x, \dot{x}, t)$ where x is the displacement of the particle at time (t). Obtain expressions for $\dot{x}(t)$ for the following cases. Given that the particle initial displacement and velocity are x_0 and v_0 respectively.

i) when the acceleration, $a = c$ and c is a constant.
ii) when the acceleration, $a = f(t)$ (i.e., function of time only).
iii) when the acceleration, $a = f(x)$ (i.e., function of displacement only).

[6.0 Marks]

b) The water sprinkler shown in Figure Q1(b), positioned at the base of a hill (A) that is defined by the equation $y = 0.5x^2$ m. Sprinkler releases a stream of water with a velocity of 15 m/s. Neglecting the size of the sprinkler, determine the followings.

i) The point $B(x, y)$ where the water strikes the ground on the hill.
ii) The time it takes from A to B.

[6.0 Marks]

Q2 a) The position of a point P in space is shown in Figure Q2(a). The distance measured along the path is $\boldsymbol{s}$ and the position vector of the particle is given by $\boldsymbol{r} = \underline{r}(t)$.

i) Define normal and tangential coordinate system to represent particle motion using unit vectors along normal direction $(\underline{e}_n)$ and tangential direction $(\underline{e}_t)$.
ii) Obtain expressions for the speed and the velocity of the particle using $\underline{e}_n$ and $\underline{e}_t$.
iii) Obtain expressions for total the acceleration of the particle using $\underline{e}_n$ and $\underline{e}_t$ and find its magnitude and directions.

[7.0 Marks]

b) As shown in Figure Q2(b), the motorcycle is traveling on a curvilinear path with the radius of curvature 150 m (measured from the motor cycle level). It was recorded that the speed of the motor cycle when it is at A is 40 m/s. The speed is then decreased at the rate of $0.05s$ ms^{-2}, where s is in meters measured from A. Determine the followings when it reaches B.

i) Speed and velocity.

ii) Acceleration.

[5.0 Marks]

Q3 a) Consider a particle moving in a curvilinear path as shown in Figure Q3(a). The particle is at A at a particular instant of time. Position vector of the particle is given by $\underline{r}$ and that makes angle θ at the given instant. The unit vectors along the radial direction and transvers direction are $\underline{e}_r$ and $\underline{e}_\theta$ respectively.

i) Write down an expression for the position vector.

ii) Determine the velocity vector of the particle and identify its radial (V_r) and transverse (V_θ) components.

iii) Determine the acceleration vector of the particle and identify its radial (a_r) and transverse (a_θ) components.

[7.0 Marks]

b) In a telescopic mechanism (Figure Q3(b)), the inner cylinder slides within the outer cylinder, which rotates about a fixed axis through the pivoting point O. If the position of the point A that is on the inner cylinder is given as $r = 2 + \sqrt{t}$ m and the angular displacement of outer cylinder is given as $\theta = \frac{t^2}{2} + 2$ rad. The time t is measured in seconds. Determine the following at $t = 2$s.

i) The total velocity of point A and its direction.

ii) The total acceleration of point A and its direction.

[5.0 Marks]

Q4 a) i) State the Newton's second law of motion for system of particles in rectilinear motion.

ii) Hence, derive a relationship for the principle of linear impulse and momentum for system of particles. Define all the terms in the relationship.

iii) What do you mean by the conservation of linear momentum? Derive it mathematically and provide one example.

[6.0 Marks]

b) A motor with a pulley system is shown in Figure Q4(b). The motor exerts a force F on the crate(B) with mass of 40 kg given by the function of time t,

$$F(t) = \begin{cases} 150 + 50t, & t < 6 \\ 450, & t \geq 6 \end{cases}$$

The maximum power of the motor is 5 kW and the motor pulley radius is 5 cm. Given that the crate moving downward at 10 m/s when $t = 0$. Determine the followings.

i) The time at which the crate changes its direction of motion.
ii) The speed and direction of the crate at $t = 6$ s.
iii) The mechanical efficiency of the motor at $t = 6$ s.

[6.0 Marks]

Q5 a) Define the terms conservative force and nonconservative force. Provide three examples for each type.

[2.0 Marks]

b) States the following theories using mathematical expressions and define each term of your expression.

i) Principle of kinetic energy and work.
ii) Conservation of energy.

[3.0 Marks]

c) Figure Q5(c) shows a linear spring with stiffness, k N/m attached to a block of mass, m kg. Suppose that the contact surfaces are rough having coefficient of friction, μ. Determine the maximum extension of the spring.

[5.0 Marks]

d) Giving an example, explain how the analysis of dynamics of a particle is differed from dynamics of a rigid body.

[2.0 Marks]

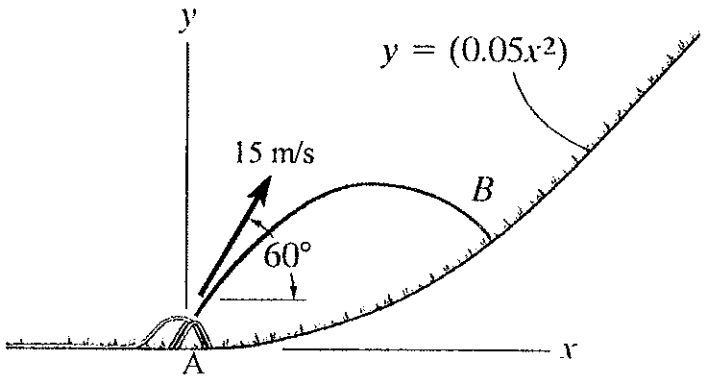

Figure Q1(b)

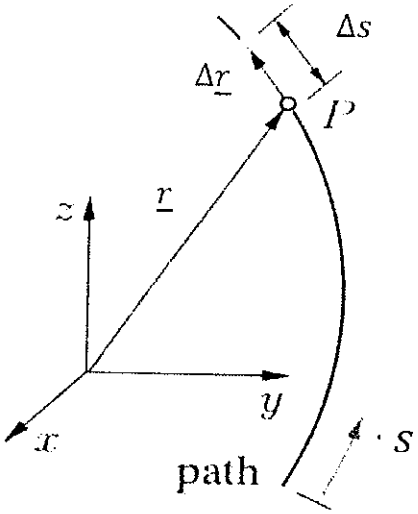

Figure Q2(a)

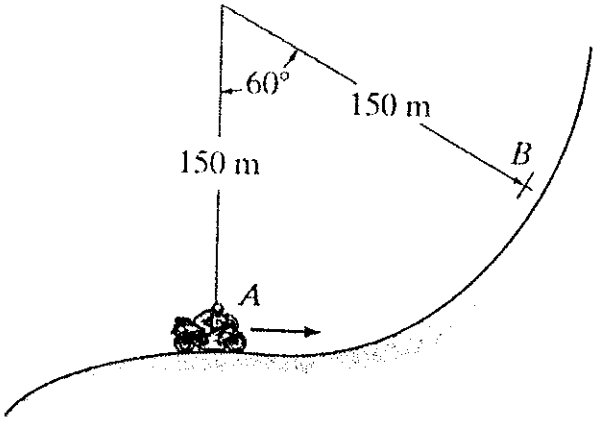

Figure Q2(b)

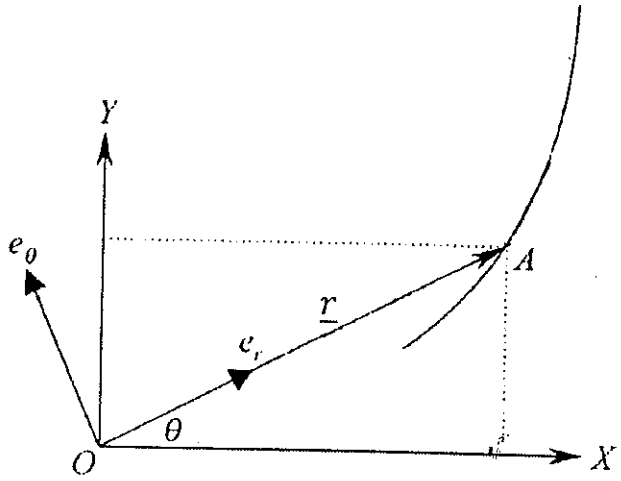

Figure Q3(a)

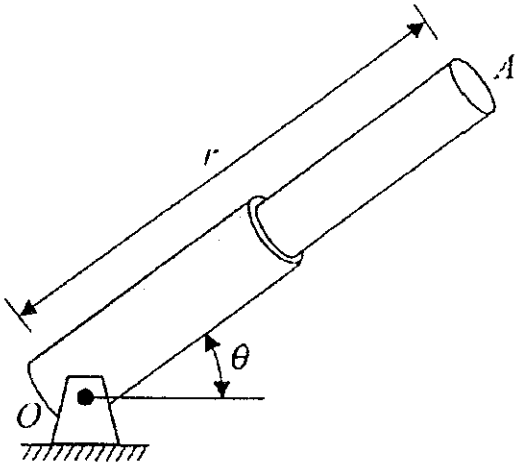

Figure Q3(b)

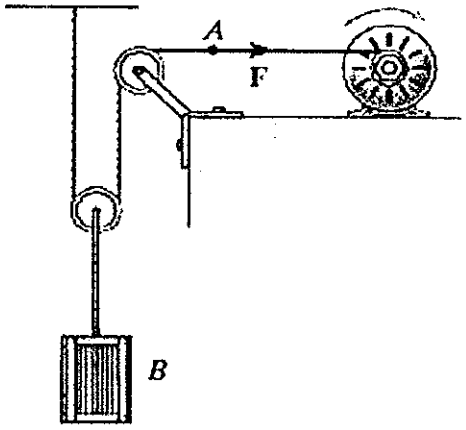

Figure Q4(b)

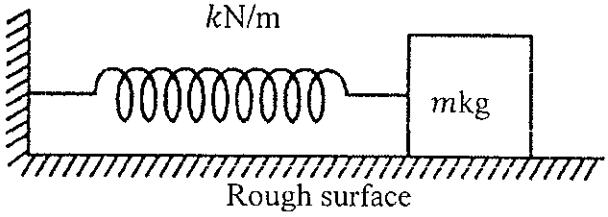

Figure Q5(c)