Library Copy

University of Ruhuna- Faculty of Technology

Bachelor of Information and Communication Technology Honours

Level 1 (Semester II) Examination, November 2022

Course Unit: TMS1233 Discrete Mathematics **Duration: 3 hours**

INSTRUCTIONS TO CANDIDATES:

- This paper contains **6 QUESTIONS** in **06 PAGES** including this sheet.
- **ANSWER ALL QUESTIONS.** All questions carry equal marks.
- All symbols have their usual meanings.
- This is a closed book examination.
- If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state it on the script.
- All Examinations are conducted under the rules and regulations of the University.

1)

1.1) Define the term 'Proposition' and state **three (3)** logical operators used to combine propositions.

(20 marks)

1.2) Let p, q, and r be the following propositions:

p: You get good marks for continuous assessments.
q: You do every exercise in the reference book.
r: You get an A for the module.

Write the following propositions using p, q, and r and logical connectives.

a) You get an A for the module, but you do not do every exercise in the reference book.

b) To get an A for the module, it is necessary for you to get good marks for continuous assessments.

c) You get good marks for continuous assessments, but you don't do every exercise in the reference book.

d) Getting good marks for continuous assessments and doing every exercise in the reference book is sufficient for getting an A for the module.

e) You will get an A for the module if and only if you either do every exercise in the reference book or you get good marks for the continuous assessments.

(35 marks)

1.3) State the **converse**, **contrapositive**, and **inverse** of the following conditional statement:

"I come to class whenever there is going to be a quiz."

(30 marks)

1.4) Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 0110110110 and 1100011101.

(15 marks)

2)

2.1) Explain the difference among Tautologies, Contradictions and Contingencies (use some examples for your explanation).

(15 marks)

2.2) Using truth tables, show that $\boldsymbol{p \rightarrow q}$ and $\boldsymbol{\neg p \vee q}$ are logically equivalent.

(15 marks)

2.3) Show that $\neg(\boldsymbol{p} \vee (\neg \boldsymbol{p} \wedge \boldsymbol{q}))$ and $\neg \boldsymbol{p} \wedge \neg \boldsymbol{q}$ are logically equivalent by developing a series of logical equivalences. (Note: Do not use a truth table to establish this equivalence). **Hence** show that $\neg(\boldsymbol{p} \rightarrow \boldsymbol{q})$ and $\boldsymbol{p} \wedge \neg \boldsymbol{q}$ are logically equivalent.

(40 marks)

2.4) a) Express the two Distributive Laws in logical equivalences.

(10 marks)

b) Determine the satisfiability of the following compound propositions and justify your answer. (Hint: you can use truth tables)

i. $(\boldsymbol{p} \vee \neg \boldsymbol{q}) \wedge (\boldsymbol{q} \vee \neg \boldsymbol{r}) \wedge (\boldsymbol{r} \vee \neg \boldsymbol{p})$

ii. $(\boldsymbol{p} \vee \boldsymbol{q} \vee \boldsymbol{r}) \wedge (\neg \boldsymbol{p} \vee \neg \boldsymbol{q} \vee \neg \boldsymbol{r})$

(20 marks)

3)

3.1) What is the truth value of $\forall x P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4? Explain your Answer.

(20 marks)

3.2) Determine the truth value (True/False) of each of the following statements if the domain consists of all real numbers. Provide explanations using examples.

a) $\exists x\ (x^3 = -1)$

b) $\exists x\ (x^4 < x^2)$

c) $\forall x\ ((-x)^2 = x^2)$

d) $\forall x\ (2x > x)$

(25 marks)

3.3) Let P(x) be the statement "x can play cricket" and let Q(x) be the statement "x speaks Tamil". Express each of the following sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can play cricket and who speaks Tamil.

b) There is a student at your school who can play cricket but who doesn't speak Tamil.

c) Every student at your school either can play cricket or speaks Tamil.

d) No student at your school can play cricket or speaks Tamil.

(20 marks)

3.4) Let N(x) be the statement "x has visited Japan" where the domain consists of the students in your school. Express each of the following quantifications in English.

a) $\exists x N(x)$

b) $\forall x N(x)$

c) $\neg \exists x N(x)$

d) $\exists x \neg N(x)$

e) $\neg \forall x N(x)$

(35 marks)

4)

4.1) Translate , the equality, $1 \cdot 0 + \overline{(1 + 0)} = 0$ into a logical equivalence.

(15 marks)

4.2) Prove the **absorption law,** $x\,(x + y) = x$ using the other identities of Boolean algebra.

(25 marks)

4.3) Find the sum-of-products expansion for the following functions using truth table.

(40 marks)

a) $F\,(x, y, z) = (x + y)\,\bar{z}$

b) $F\,(x, y, z) = y(x + x\bar{y} + \bar{z})$

4.4) Construct circuits from inverters, AND gates, and OR gates to produce the following outputs. (20 marks)

a) $\bar{x}(x + y)$

b) $\bar{x}\,\overline{(y + \bar{z})}$

c) $(x + y + z)(\bar{x}\,\bar{y}\,\bar{z})$

d) $x\,y + \bar{x}\,\bar{y}$

5)

5.1) Explain the rule of inference called "Modus ponens".

(10 marks)

5.2) State which argument form (rule of inference) is the basis of the each of following arguments and use variables to represent it.

a) Elephants live in Sri Lanka and are mammals. Therefore, elephants are mammals.

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

c) Piyumi is an excellent swimmer. If Piyumi is an excellent swimmer, then she can work as a lifeguard. Therefore, Piyumi can work as a lifeguard.

d) Kasun will work at a computer company next year. Therefore, next year Kasun will work at a computer company or he will be a teacher.

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

(50 marks)

5.3) Show that the premises "If you eat Kottu, then you will suffer from stomach ache," "If you do not eat Kottu, then you will sleep well," and "If you sleep well, then you will wake up feeling refreshed" lead to the conclusion "If you do not eat Kottu, then you will wake up feeling refreshed."

(20 marks)

5.4) Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

(20 marks)

6)

6.1) Find the cardinal number of the following sets. (15 marks)

a) A = {x: x ∈ Z, 2 < x < 7}

b) B = {∅, {∅}, {∅, {∅}}}

c) C = {x: x ∈ Z+, x < 100}

6.2) Write down the power set of the following sets. (15 marks)

a) A = {0, 1, 2}

b) B = {∅}

c) C = {∅, {∅}}

6.3) Determine whether each of the following statements is true or false. (35 marks)

a) 0∈∅
b) ∅∈{0}
c) {0}⊂∅
d) ∅⊂{ 0}
e) {0}∈{ 0}
f) {0}⊂{ 0}
g) {∅}⊆{∅}

6.4) a) Construct a membership table to show that the distributive law holds: (15 marks)

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

b) Use set builder notation and logical equivalences to establish the first De Morgan law: (20 marks)

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$