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University of Ruhuna - Faculty of Technology

Bachelor of Engineering Technology and Information & Communication Technology

Level II (Semester II) Examination, November/December 2022

Course Unit: TMS2213 Probability and Statistics

Time Allowed Two (02) hours

Answer all **Five (05)** questions. Calculators are allowed to use for calculations.

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1. The lengths of power failures, in minutes, are recorded in the following table.

22	18	35	15	29	42	69	38	10	13
55	28	12	15	13	22	14	11	30	36
24	19	13	24	21	12	21	40	98	37

(a) Construct a stem-and-leaf diagram for these data.

(10 marks)

(b) Find the sample mean and sample median of the power-failure times.

(20 marks)

(c) Find the sample standard deviation of the power failure times.

(10 marks)

(d) Find the value of first quartile Q_1 and third quartile Q_3 and the IQR for the sample.

(25 marks)

(e) Construct a box-and-whisker plot for these data.

(25 marks)

(f) Discuss the shape of the distribution of the power-failure times.

(10 marks)

2.

(a) Military radar and missile detection systems are designed to warn a country of an enemy attacks. A reliability question is whether a detection system will be able to identify an attack and issue a warning. Assume that a particular detection system has a 0.9 probability of detecting a missile attack. Use the binomial probability distribution to answer the following questions.

(i) If two detection systems are installed in the same area and operate independently, what is the probability that at least one of the systems will detect the attack?

(20 marks)

(ii) If three systems are installed, what is the probability that at least one of the systems will .detect the attack?

(30 marks)

(b) An inventory study determines that, on average, demands for a particular item at a warehouse are made 5 times per day. What is the probability that on a given day this item is requested

(i) exactly two time? (10 marks)

(ii) not at all? (10 marks)

(iii)more than 2 times? (30 marks)

3.

(a) An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns

(i) less than 700 hours (20 marks)

(ii) between 778 and 834 hours. (30 marks)

(iii) greater than 850 hours (30 marks)

(b) The mean life of a certain brand of auto batteries is 44 months with a standard deviation of 3 months. Assume that the lives of all auto batteries of this brand have a bell-shaped distribution. Using the empirical rule, find the percentage of auto batteries of this brand that have a life of 38 to 50 months.

(20 marks)

4.

(a) What are the most suitable point estimators for the population mean, μ and the population variance σ^2?

(10 marks)

(b) The following data give the speeds (in kilometers per hour), as measured by radar, of 10 cars traveling on southern expressway.

76 72 80 68 76 74 71 78 82 65

Assuming that the speeds of all cars traveling on this highway have a normal distribution, construct a 90% confidence interval for the mean speed of all cars traveling on this highway.

(40 marks)

(c) The deflection temperature under load for two different types of plastic pipe is being investigated. Two random samples of 15 pipe specimens are tested, and the deflection temperatures observed are as follows (in ^{0}F):

Type 1: 206, 188, 205, 187, 194, 193, 207, 185, 189, 213, 192, 210, 194, 178, 205

Type 2: 177, 197, 206, 201, 180, 176, 185, 200, 197, 192, 198, 188, 189, 203, 192

Assume that the temperatures are normally distributed for each of the two types and that the standard deviations for the two populations are equal. Do the data support the claim that the deflection temperature under load for type 1 pipe is greater than that of type 2? Use α= 0.05.

(50 marks)

5.

The relationship between electricity consumption and household income was studied, yielding the following data on household income X (in units of Rs. 1000 per month) and electricity consumption Y (in units of **kilowatts**).

Household income (x)	20.0	30.5	40.0	55.1	60.3	74.9	88.4	95.2
Electricity consumption (y)	64	77	91	98	107	128	134	140

(a) Assuming the model $y = \beta_0 + \beta_1 x + \epsilon$, find the regression line with electricity consumption as a dependent variable and household income as an independent variable.

(20 marks)

(b) If x=50 (household income of Rs. 50000), estimate the average electricity consumed for households of this income.

(10 marks)

(c) How much would you expect the change in consumption to be if any household income increases Rs. 2000 per month?

(10 marks)

(d) Compute the coefficient of determination and give a brief interpretation of it.

(20 marks)

(e) Test the hypothesis that $\beta_1 = 0$. Do the results of this test indicate that a linear trend is significant?

(40 marks)

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Supplementary Statistical Formulas

- Mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- Variance

$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2\right] = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right] \qquad \sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{N}(x_i - \mu)^2\right] = \frac{1}{N}\left[\sum_{i=1}^{N} x_i^2 - N\mu^2\right]$$

- For event A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Standardizing a normal random variable ($X \sim N(\mu, \sigma^2)$)
  $z = \frac{X-\mu}{\sigma}$

- If $X_1, \cdots, X_n$ are independent and identically distributed random varibles and $X_i \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \sigma^2/n)$, where $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$

- Tests for a hypothesized population mean (μ_0). Under H_0

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1) \qquad \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

- $(1-\alpha)100\%$ Confidence interval for the population mean

$$\left(\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) \qquad \left(\bar{x} - t_{n-1,\frac{\alpha}{2}}\frac{s}{\sqrt{n}}, \bar{x} + t_{n-1,\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\right)$$

- The sample size required to estimate μ with a $(1-\alpha)100\%$ confidence interval:

$$n = \left[\frac{z_{\frac{\alpha}{2}}\sigma}{B}\right]^2$$

- Testing for a difference in population means for two independent populations, under H_0,

$$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

$$\frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{1/n_1 + 1/n_2}} \sim t_{n_1+n_2-2} \text{ where } s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t_\nu \text{ where } \nu = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

- Testing for a difference in population means for two dependent populations. Under H_0,

$$\frac{\bar{d}}{s_d/\sqrt{n}} \sim t_{n-1}$$

where $\bar{d}$-Mean of paired differences, s_d-Standard deviation of paired differences

- Correaltion
Pearson's correaltion coefficient (r) between X and Y

$$r = \frac{\sum_i x_i y_i - n\bar{x}\bar{y}}{\sqrt{(\sum_i x_i^2 - n\bar{x}^2)(\sum_i y_i^2 - n\bar{y}^2)}}$$

Test the significance of the correlation coefficient (ρ). Under H_0,

$$\frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2}$$

- Regression
Estimated regression coefficients
Slope

$$\hat{\beta}_1 = b_1 = \frac{\sum_i x_i y_i - n\bar{x}\bar{y}}{\sum_i x_i^2 - n\bar{x}^2}$$

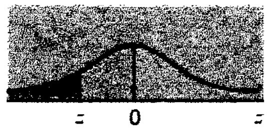
Intercept

$$\hat{\beta}_0 = b_0 = \bar{y} - b_1\bar{x}$$

Residual(e_i)=$y_i - \hat{y}_i$
Sum of Squares Regression(SSR) $=\sum_i(\hat{y}_i - \bar{y})^2$ df=1
Sum of Squares Error (SSE) $= \sum_i(y_i - \hat{y}_i)^2$ df=$n-2$
Sum of Squares Total (SST) $= \sum_i(y_i - \bar{y})^2$ df=$n-1$
SST=SSR+SSE F=MSR/MSE$\sim F_{1,n-2}$ $R^2 = \frac{SSR}{SST}$
$SS_{xx} = \sum x_i^2 - n\bar{x}^2$ $SST = SS_{yy} = \sum y_i^2 - n\bar{y}^2$ $SS_{xy} = \sum x_i y_i - n\bar{x}\bar{y}$
$SSE = SS_{yy} - b_1 SS_{xy}$
$S_\epsilon = \sqrt{\frac{SSE}{n-2}}$
$S_{b_1} = \frac{S_\epsilon}{\sqrt{SS_{xx}}}$
$t = \frac{b_1}{S_{b_1}} \sim t_{n-2}$

Table IV Standard Normal Distribution Table

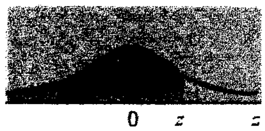
The entries in this table give the cumulative area under the standard normal curve to the left of z with the values of z equal to 0 or negative.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table IV Standard Normal Distribution Table (continued)

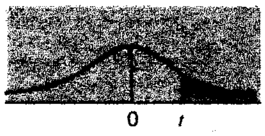
The entries in this table give the cumulative area under the standard normal curve to the left of z with the values of z equal to 0 or positive.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table V The *t* Distribution Table

The entries in this table give the critical values of *t* for the specified number of degrees of freedom and areas in the right tail.



df	Area in the Right Tail Under the *t* Distribution Curve					
	.10	**.05**	**.025**	**.01**	**.005**	**.001**
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
31	1.309	1.696	2.040	2.453	2.744	3.375
32	1.309	1.694	2.037	2.449	2.738	3.365
33	1.308	1.692	2.035	2.445	2.733	3.356
34	1.307	1.691	2.032	2.441	2.728	3.348
35	1.306	1.690	2.030	2.438	2.724	3.340

Table V The *t* Distribution Table (continued)

	Area in the Right Tail Under the *t* Distribution Curve					
df	.10	.05	.025	.01	.005	.001
36	1.306	1.688	2.028	2.434	2.719	3.333
37	1.305	1.687	2.026	2.431	2.715	3.326
38	1.304	1.686	2.024	2.429	2.712	3.319
39	1.304	1.685	2.023	2.426	2.708	3.313
40	1.303	1.684	2.021	2.423	2.704	3.307
41	1.303	1.683	2.020	2.421	2.701	3.301
42	1.302	1.682	2.018	2.418	2.698	3.296
43	1.302	1.681	2.017	2.416	2.695	3.291
44	1.301	1.680	2.015	2.414	2.692	3.286
45	1.301	1.679	2.014	2.412	2.690	3.281
46	1.300	1.679	2.013	2.410	2.687	3.277
47	1.300	1.678	2.012	2.408	2.685	3.273
48	1.299	1.677	2.011	2.407	2.682	3.269
49	1.299	1.677	2.010	2.405	2.680	3.265
50	1.299	1.676	2.009	2.403	2.678	3.261
51	1.298	1.675	2.008	2.402	2.676	3.258
52	1.298	1.675	2.007	2.400	2.674	3.255
53	1.298	1.674	2.006	2.399	2.672	3.251
54	1.297	1.674	2.005	2.397	2.670	3.248
55	1.297	1.673	2.004	2.396	2.668	3.245
56	1.297	1.673	2.003	2.395	2.667	3.242
57	1.297	1.672	2.002	2.394	2.665	3.239
58	1.296	1.672	2.002	2.392	2.663	3.237
59	1.296	1.671	2.001	2.391	2.662	3.234
60	1.296	1.671	2.000	2.390	2.660	3.232
61	1.296	1.670	2.000	2.389	2.659	3.229
62	1.295	1.670	1.999	2.388	2.657	3.227
63	1.295	1.669	1.998	2.387	2.656	3.225
64	1.295	1.669	1.998	2.386	2.655	3.223
65	1.295	1.669	1.997	2.385	2.654	3.220
66	1.295	1.668	1.997	2.384	2.652	3.218
67	1.294	1.668	1.996	2.383	2.651	3.216
68	1.294	1.668	1.995	2.382	2.650	3.214
69	1.294	1.667	1.995	2.382	2.649	3.213
70	1.294	1.667	1.994	2.381	2.648	3.211
71	1.294	1.667	1.994	2.380	2.647	3.209
72	1.293	1.666	1.993	2.379	2.646	3.207
73	1.293	1.666	1.993	2.379	2.645	3.206
74	1.293	1.666	1.993	2.378	2.644	3.204
75	1.293	1.665	1.992	2.377	2.643	3.202
∞	1.282	1.645	1.960	2.326	2.576	3.090