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An unbiased GM(1,1)-based new hybrid approach for time series forecasting

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Abstract

Purpose – The time series forecasting is an essential methodology which can be used for analysing time series data in order to extract meaningful statistics based on the information obtained from past and present. These modelling approaches are particularly complicated when the available resources are limited as well as anomalous. The purpose of this paper is to propose a new hybrid forecasting approach based on unbiased GM(1,1) and artificial neural network (UBGM_BPNN) to forecast time series patterns to predict future behaviours. The empirical investigation was conducted by using daily share prices in Colombo Stock Exchange, Sri Lanka.

Design/methodology/approach – The methodology of this study is running under three main phases as follows. In the first phase, traditional grey operational mechanisms, namely, GM(1,1), unbiased GM(1,1) and nonlinear grey Bernoulli model, are used. In the second phase, the new proposed hybrid approach, namely, UBGM_BPNN was implemented successfully for forecasting short-term predictions under high volatility. In the last stage, to pick out the most suitable model for forecasting with a limited number of observations, three model-accuracy standards were employed. They are mean absolute deviation, mean absolute percentage error and root-mean-square error.

Findings – The empirical results disclosed that the UNBG_BPNN model gives the minimum error accuracies in both training and testing stages. Furthermore, results indicated that UNBG_BPNN affords the best simulation result than other selected models.

Practical implications – The authors strongly believe that this study will provide significant contributions to domestic and international policy makers as well as government to open up a new direction to develop investments in the future.

Originality/value – The new proposed UBGM_BPNN hybrid forecasting methodology is better to handle incomplete, noisy, and uncertain data in both model building and *ex post* testing stages.

Keywords GM(1,1), NGBM, Time series forecasting, UNBG_BPNN, Unbiased GM(1,1)

Paper type Research paper

1. Introduction

The time series forecasting is an essential tool which can be used for analysing time-related data in order to extract meaningful statistics as well as characteristics based on the information obtained from past and present. These modelling approaches are

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particularly complicated when limited resources are available during the underlying data gathering process. Currently, the time series forecasting approaches are widely used in the field of finance, applied sciences, engineering and so on for data processing, pattern recognition, forecasting, etc. As a result of the importunacy as well as applicability, much effort has been given by scholars over the past decades to introduce and improve novel time series forecasting models based on the miscellaneous type of mathematical as well as computational assumptions.

According to the literature, miscellaneous methodologies are available under different frameworks. Basically, these existing methodologies can be listed under two main categories: frequent domain and time domain. However, most of the forecasting approaches have been developing in the time domain framework under different categories such as parametric vs non-parametric, classical vs statistics and linear vs nonlinear. Among them, stationary stochastic process-based parametric approaches have been dominating many areas of forecasting literature. For example, the well-balanced statistical assumptions with Box-Jenkins models, auto regressive, moving average, autoregressive moving average, and its generalization models of autoregressive integrated moving average (ARIMA) are significant. However, most of these traditional approaches are more suitable and appropriate only for empirical data studies under the normality, linearity and stationary assumptions.

Because of the poor forecasting abilities to deal with the uncertainties, fuzzy, insufficient and high-volatility data patterns, much more effort has been expended by scholars to develop miscellaneous mechanisms in the past decades for forecasting time series data patterns. These include the artificial neural network (ANN) to predict real-world data applications (Liu *et al.*, 2013; Egrioglu *et al.*, 2014; Claveria and Torra, 2014; Laboissiere *et al.*, 2015; Feng *et al.*, 2015; Adhikari, 2015; Günay, 2016); Panapakidis and Dagoumas, 2016, the integration of fuzzy logistics techniques-based artificial intelligence systems for forecasting (Inman *et al.*, 2013; Gunasekaran and Ngai, 2014; Garrido *et al.*, 2015; Camacho-Collados and Liberatore, 2015; Agrawal *et al.*, 2015), Box and Jenkins statistical approaches for forecasting economic and financial indices (Wedding and Cios, 1996; Lu and AbouRizk, 2009; Hamzaçebi *et al.*, 2009; Du *et al.*, 2014; Aboagye-Sarfo *et al.*, 2015; Aizenberg, 2016), grey system theory (Li *et al.*, 2012; Lei and Feng, 2012; Jin *et al.*, 2012), and so on.

As a result of the high-volatile chaotic behaviour in real-world applications, the time series forecasting by using separate linear and nonlinear models is always considered to be a difficult and challenging task in the multidisciplinary systems. As a result of these complications with regards to the traditional time series approaches, neural network computing model-based new hybrid methodologies were introduced and developed by McCulloch and Pitts to handle incomplete, noisy and uncertain data (Warren and Walter, 1943; Zhang, 2003).

In this scenario, contribution by Zhang *et al.* is significant (Zhang, 2003). For the very first time in literature, they have proposed a new hybrid time series forecasting approach that combines both ARIMA and ANN mechanisms to take extra benefit of the unique strengths of linear and nonlinear domains. Because of the high-flexibility nonlinear modelling capability, this novel concept has been successfully applied in various systems such as finance and economic (Khashei and Bijari, 2011; Chai and Lim, 2016; Adhikari, 2015; Lahmiri, 2016), energy (Azimi *et al.*, 2016; Deb *et al.*, 2015; Günay, 2016; Lou and Dong, 2015; Men *et al.*, 2016), geological systems (Babkin *et al.*, 2015; Durdu, 2010; Günay, 2016; Khashei and Bijari, 2011; Wang *et al.*, 2016), agricultural systems (Mo *et al.*, 2016) and other systems (Sánchez Lasheras *et al.*, 2015; Wang *et al.*, 2015) for forecasting time series data patterns; especially, this concept is more suitable and appropriate for forecasting stock market predictions under the nonlinear high-volatility condition.

Compared with the long-term forecasting with huge sampling frequencies, the development of short-term forecasting has been still in their infancy. In this scenario, grey system theory and its generalized mechanisms were introduced and propounded in the early 1980s by a famous Chinese Scholar Deng Ju-long (Zhou and He, 2013). Since then, this new mechanism has been applied in many fields such as industry (Bahrami *et al.*, 2014; Hamzacebi and Es, 2014; Lei and Feng, 2012), manufacturing (Chang *et al.*, 2015; Wang *et al.*, 2013), natural sciences (Intharathirat *et al.*, 2015), transportations (Jiang *et al.*, 2014), IT (Wang, 2013; Wang *et al.*, 2014), etc.

Among the various grey methods, the GM(1,1) is suitable and widely used in the literature. This model is basically developed to fit non-negative raw data with exponential form without any irreducible volatility patterns (Adhikari, 2015; Kayacan *et al.*, 2010). Furthermore, GM(2,1), grey Verhulst (Xu *et al.*, 2011) and nonlinear grey Bernoulli models (NGBM) (Chen *et al.*, 2010) have been developed for analysing the data with oscillatory, "S" distribution, and nonlinear data patterns, respectively.

To fulfil these limitations, which were listed in the literature, this current study developed a new hybrid forecasting methodology based on unbiased GM(1,1) and feed-forward back propagation neural network (BPNN) to handle incomplete, noisy and uncertain data. For comparative purpose, GM(1,1), unbiased GM(1,1), and NGBM forecasting approaches are used. The empirical investigation of the proposed new hybrid method is conducted by using daily share prices in the Colombo Stock Exchange (CSE), Sri Lanka. We strongly believed that this proposed mechanism can be widely applied for identifying the meaningful characteristics to make future adjustments in the fields of science, finance and engineering, in both theoretical and practical perspectives.

The remainder of this study is organized as follows. In Section 2 and Section 3, the theoretical background of grey theory and the new proposed hybrid model are discussed, respectively. The empirical results with model comparisons are discussed in Section 4. Section 5 gives the discussion and ends up with the conclusion, policy issues and future work.

2. Methodology

The grey system theory was brought forward and developed by Professor Deng Ju-long for dealing with systems with a limited number of data modelling. Currently, grey model applications have been successfully applied in different fields for analysing data with poor information. In the current study, we have focussed on the basic principles and modelling process of GM(1,1), unbiased GM(1,1) and NGBM models.

2.1 Overview of grey models accumulation: GM(1,1) model

Among the miscellaneous grey mechanisms, GM(1,1) is significantly used with a certain degree of accuracy. The modelling approach goes through the steps wisely as follows (Ji and Zhang, 2011; Rathnayaka *et al.*, 2015):

- Step 1: develop a conceptual modeling language for through and concepts.
- Step 2: determine the factors.
- Step 3: examine the causal relations among the factors which have been identified in Step 2.

- Step 4: identify the most suitable inputs and outputs to build a dynamic model.
- Step 5: evaluate the model accuracies and make conclusions.

Based on these steps, the first-order GM(1,1) modelling algorithm goes through the following events (Chen *et al.*, 2010).

Let $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots\ldots\ldots, x^{(0)}(n))$ be a sequence of data, which are taken in consecutive order at equal time intervals:

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ ....,\ x^{(0)}(n) \middle| n \geqslant 4\right\}$$

Assume that $X^{(0)}$ is non-negative raw data series, where an original series of raw data contains n entries.

Theorem 1. (Julong, 1989): accumulated generating operation (AGO) generating datum bound comprises both raw component as follows:

$$x^{(1)}(k) = x^{(0)}(k) + x^{(1)}(k-1)$$

raw part generated part

Theorem 2. (Julong, 1989): let $x^{(0)}$ be a raw series:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ ....,\ x^{(0)}(n)\right)$$

y is said to be inverse accumulated generating operation (IAGO) series of $x^{(0)}$ where:

$$y = IAGOx^{(0)}$$

provided that:

$$y = (y(1), y(2), \ ...,\ y(n))$$

for $(k) \in y$, satisfies that:

$$y(k) = x^{(0)}(k) - x^{(0)}(k-1)$$

Step 1 (Zhou and He, 2013; Zhong *et al.*, 2011): according to Theorem 1, the AGO, which implies adding raw data in series of $X^{(0)}$, is given by the following:

$$\text{AGO}\left(x^{(0)}\right): X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ \ldots\ldots\ldots,\ x^{(1)}(n)\right\}$$

where $X^{(1)}$ is an AGO series, which represents the first-order accumulated generating operator, given by the following:

$$x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m), \quad k = 1, 2, \ ...,\ n, \quad n \geqslant 4 \tag{1}$$

the $X^{(0)}$ is the first-order inverse accumulating generation sequence of $X^{(1)}$.

Where:

$$X^{(0)}(k) = X^{(1)}(k) - x^{(1)}(k-1), \;\; k = 2, 3, \ldots\ldots\ldots, n.$$

As an initial condition, $X^{(0)}(1) = X^{(0)}(1)$.

Step 2: the mean generating operation (MEAN$(X)^{(1)}$)) denotes the averaging adjoining data in mean consecutive neighbours, generating operator for $X^{(1)}$:

$$\mathrm{MEAN}\left(X^{(1)}\right): Z^{(1)} = \left\{z^{(1)}(1),\ z^{(1)}(2),\ \ldots\ldots\ldots,\ z^{(1)}(n)\right\}$$

where:

$$Z^{(1)}(k) = \mathrm{MEAN}\,X^{(1)} = \frac{1}{2}\left(X^{(1)}(k) + X^{(1)}(k-1)\right); \quad k = 2,\ 3,\ \ldots,\ n, \quad n \geqslant 4 \quad (2)$$

The least-square estimator (LST) of the grey difference equation of GM(1,1) is defined as follows in equation:

$$x^{(0)}(k) + az^{(1)}(k) = b \quad k = 2, 3,\ \ldots,\ n \quad (3)$$

where undetermined parameters a and b are called developmental coefficient and grey input, respectively. Furthermore, the $z^{(1)}(k)$ is said to be mean series of $x^{(1)}(k)$.

Step 3: theoretically, AGO($x^{(0)}$) represent monotonic increase series, which represents the behaviours of the first-order differential equation. Therefore, the whitenization equation of the first-order differential equation is defined as follows:

$$\frac{d\hat{x}^{(1)}}{dt} + a\hat{x}^{(1)} = b \quad (4)$$

where both a and b are the interim parameters (developmental coefficient) of prediction values of the grey model, respectively.

The derivatives of the function $\hat{x}^{(1)}$ at t can be defined as follows:

$$\frac{\frac{d\hat{x}^{(1)}}{dt} = \lim\limits_{\Delta t \to 0} \hat{x}^{(1)}(t+\Delta t) - \hat{x}^{(1)}(t)}{\Delta t}$$

If the sampling time interval Δt is unit, then we can assume that, $\Delta t \to 1$. So equation can be reduced as follows:

$$\frac{d\hat{x}^{(1)}}{dt} \cong x^{(1)}(k+1) - x^{(1)}(k); \quad k = 1, 2, 3,\ \ldots,\ n \quad (5)$$

Step 4: to estimate the developing coefficient of grey inputs a and b, the LSTs with augmented matrix can be obtained as follows:

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

where $Y_n = BU$, $\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (B^T B)^{-1} B^T Y$, and $U = \begin{pmatrix} a \\ b \end{pmatrix}$.

Where B implies the accumulated matrix and Y denotes the constant vector.

Step 5: solving the grey reflection Equation (5) from Step 3, the particular solution of the AGO grey prediction can be approximate as follows:

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-a(k-1)} + \frac{b}{a}; \quad k = 1, 2, ..., n \tag{6}$$

where $x^{(0)}(1) = \hat{x}^{(0)}(1)$.

Step 6: to obtain the predicted values of the primitive data at time (k+1), substituting AGO (IAGO) operator from Step 2 with Theorem 2, the simulation function of $\hat{x}^{(0)}(k+1)$ can be obtained as follows:

$$\hat{x}^{(0)}(k+1) = (1 - e^{-a}) \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}; \quad k = 1, 2, \ldots, n \tag{7}$$

where $\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), ..., \hat{x}^{(0)}(n)$ and $\hat{x}^{(0)}(n+1), \hat{x}^{(0)}(n+2), \ldots$ are GM(1,1) fitted values and forecasted values, respectively.

2.2 Unbiased GM(1,1) model

Based on the empirical studies which have been done based on GM(1,1), it can be suggested that GM(1,1) has been giving high-accurate results only with raw data sequences with slow mutations (Peirong *et al.*, 2008; Ji *et al.*, 2007). Under this scenario, Ji *et al.* modified traditional GM(1,1) and proposed an unbiased GM(1,1) methodology to overcome these limitations (Ji *et al.*, 2007, 2010).

The modelling steps of the unbiased GM(1,1) method run as follows (Peirong *et al.*, 2008):

Step 1: define the first-order AGO on $X^{(0)}, x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m), k = 1, 2, \ldots, n$, $n \geqslant 4$.

Step 2: create a data matrix B, Y:

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \quad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

where $Z^{(1)}(k) = 1/2(X^{(1)}(k) + X^{(1)}(k-1)); k = 2, 3, ..., n, \quad n \geqslant 4$ and $k \neq 1$

Step 3: create accumulated matrix:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left(B^T B\right)^{-1} B^T Y$$

Step 4: create parameters of unbiased GM(1,1):

$$a' = \ln \frac{2 - \hat{a}}{2 + \hat{a}} \quad A' = \ln \frac{2\hat{b}}{2 + \hat{a}}$$

Step 5: establish the model of raw data sequence $x^{(0)}$:

$$\hat{X}^{(0)}(1) = X^{(0)}(1)$$

$$\hat{X}^{(0)}(k) = A' e^{a'(k-1)}; \quad k = 2, 3, \ldots, n \quad n \geqslant 4 \text{ and } k \neq 1$$

2.3 NGBM

Among the numerous types of forecasting methodologies, the NGBM is one of the significant methodologies that can be widely used for handling uncertain systems with limited information. The NGBM methodology is constructed as follows:

Assume that the original non-negative original data sequence with n entries is given by the following:

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ldots\ldots\ldots, x^{(0)}(n) \middle| n \geqslant 4\right\}.$$

where $x^{(0)}(k)$ is the value of the behaviour series at k, $k = 1, 2, \ldots, n$.

Steps 1 and 2 are similar to GM(1,1).

Step 3: define the grey differential equation of the NGBM(1,1)

The whitenized differential equation of NGBM(1,1) has the following form:

$$x^{(0)}(k) + az^{(1)}(k) = b\left[z^{(1)}(k)\right]^{\gamma} k = 2, 3, \ \ldots, \ n \text{ and } \gamma \neq \tag{8}$$

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b\left(x^{(1)}\right)^{\gamma} \tag{9}$$

where $x^{(0)}(k)$ is a grey derivative, a and b are unknown model parameters, and $Z^{(1)}(k) = 1/2(X^{(1)}(k) + X^{(1)}(k-1))$; $k = 2, 3, \ \ldots, \ n, \quad n \geqslant 4$ is referred to the background values of grey derivatives. Furthermore, γ is the power exponent which belongs to the real values excluding 1.

Lemma 1. To choose optimal power exponent. The power exponent γ is presented as follows:

$$\gamma = \frac{\mathbf{1}}{\boldsymbol{n-2}} \sum_{\boldsymbol{k=2}}^{\boldsymbol{n-1}} \gamma(\boldsymbol{k})$$

where:

$$\gamma(\boldsymbol{k}) = \frac{\left\{\left[\boldsymbol{X}^{(0)}(\boldsymbol{k}+1) - \boldsymbol{X}^{(0)}(\boldsymbol{k})\right].\boldsymbol{Z}^{(1)}(\boldsymbol{k}+1).\boldsymbol{Z}^{(1)}(\boldsymbol{k}).\boldsymbol{X}^{(0)}(\boldsymbol{k}) - \left[\boldsymbol{X}^{(0)}(\boldsymbol{k}) - \boldsymbol{X}^{(0)}(\boldsymbol{k}-1)\right].\boldsymbol{Z}^{(1)}(\boldsymbol{k}+1).\boldsymbol{Z}^{(1)}(\boldsymbol{k}).\boldsymbol{X}^{(0)}(\boldsymbol{k}+1)\right\}}{\left\{\left[\boldsymbol{X}^{(0)}(\boldsymbol{k}+1)\right]^2.\boldsymbol{Z}^{(1)}(\boldsymbol{k})\boldsymbol{X}^{(0)}(\boldsymbol{k}) - \left[\boldsymbol{X}^{(0)}(\boldsymbol{k})\right]^2.\boldsymbol{Z}^{(1)}(\boldsymbol{k}+1)\boldsymbol{X}^{(0)}(\boldsymbol{k}+1)\right\}}$$

$$\boldsymbol{k} = 2, 3, \ldots, n-1$$

Step 4: adopt least-square method to estimate model parameters. The system can be converted for the augmented matrix as follows:

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} -z^{(1)}(2) & \left(z^{(1)}(2)\right)^{\gamma} \\ -z^{(1)}(3) & \left(z^{(1)}(3)\right)^{\gamma} \\ \vdots & \vdots \\ -z^{(1)}(n) & \left(z^{(1)}(n)\right)^{\gamma} \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

where $Y_n = BU$, $\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (B^T B)^{-1} B^T Y$ and $U = \begin{pmatrix} a \\ b \end{pmatrix}$.

Where B and Y imply the accumulated matrix and constant vector, respectively.

Step 5: model selection. Based on the dimensions of γ, the model selection criteria can be defined as follows:

$$\text{Grey model} = \begin{cases} \text{GM}(1,1) & \text{if } \gamma = 0 \\ \text{Grey} - \text{Verhulst} & \text{if } \gamma = 2 \\ \text{NGBM} & \text{if } \gamma \geqslant 2 \end{cases}$$

if $\gamma = 2$, it is the grey Verhulst model.

Based on the grey system methodology, the new concept was introduced by Pierre Franois Verhulst for forecasting exponential behavioural data patterns. The new methodology can be defined based on the following steps.

The time response sequence of grey Verhulst model may be written as follows:

$$\hat{x}^{(1)}(k+1) = \frac{1}{D+Ce^{ak}}$$

where $C = \left[\frac{1}{x^{(1)}(1)} - D\right]$ and $D = b/a$

If $\gamma > 2$, it is the NGBM model.

According to the first-order differential conditions, the particular solution for whitenized equation can be expressed as follows:

$$\hat{x}^{(1)}(k+1) = \left[\left(x^{(0)}(1)^{1-n} - D\right)e^{-a(1-n)k} + D\right]^{1/(1-n)} n \neq 1 \text{ and } k = 1, 2, 3, \ldots$$

Step 6: perform the IAGO. To obtain the fitted values and predicted values, the IAGO may be applied:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k); \quad k = 1, 2, 3, \ldots, n$$

$$\hat{x}^{(1)}(1) = x^{(0)}(1)$$

where $\hat{x}^{(0)}(n+1)$, $\hat{x}^{(0)}(n+2)$, $\hat{x}^{(0)}(n+3)$, … are forecasted values of the grey Verhulst model.

3. An unbiased GM(1,1)-based new hybrid approach for time series forecasting

According to the recent literature, modified unbiased GM(1,1) (MUGM) and ANN methodologies have been widely applied to achieve high-accuracy forecasting in linear and nonlinear domains. However, some of these methodologies are not fully suitable under the high-volatility irrational patterns. As a result, combined methodologies under the linear autocorrelation structure and nonlinear weighted average component have created high-accuracy forecasting than single model approaches. According to Zhang (2003), if there are any outliers or multicollinearity in the data, neural network can significantly outperform linear regression models.

The proposed methodology is considered a time series and is to be composed in two structures as linear autocorrelation structure and nonlinear component as follows (Rathnayaka *et al.*, 2015; Zhang, 2003):

$$Y_t = \alpha L_t + \beta N_t \tag{10}$$

where L_t and N_t denote the linear and nonlinear components with their coefficients of α and β, respectively. In the first phase, MUGM approach is mainly used to analyse the linear part of problem. Based on the residual analysis results, in the second stage, neural network model-based approach was applied to capture the nonlinearity.

Let us assume that the residual from the linear model will contain only the nonlinear relationships. The residuals of the linear component can be defined as follows:

$$e_t = Y_t - \widehat{L_t} \tag{11}$$

where e_t denotes the residual of linear model at time t, and $\widehat{L_t}$ presents the forecasting value for the estimated MUGM at time t. According to the results, if we can find any nonlinear significant pattern in the residuals, it indicates the limitations of MUGM, and as a next step, ANN modelling approach can be applied to discover the nonlinear relationships:

$$e_t = f(e_{t-1}, e_{t-2}, e_{t-3}, \ldots, e_{t-n}) + \varepsilon_t \tag{12}$$

where n represent the input nodes, and f is the nonlinear function, which is determined based on the ANN approach. However, if the nonlinear model is not an appropriate, it means that the error term ε_t is not necessarily random:

$$\widehat{y_t} = \widehat{L_t} + \widehat{N_t} \tag{13}$$

4. Empirical results

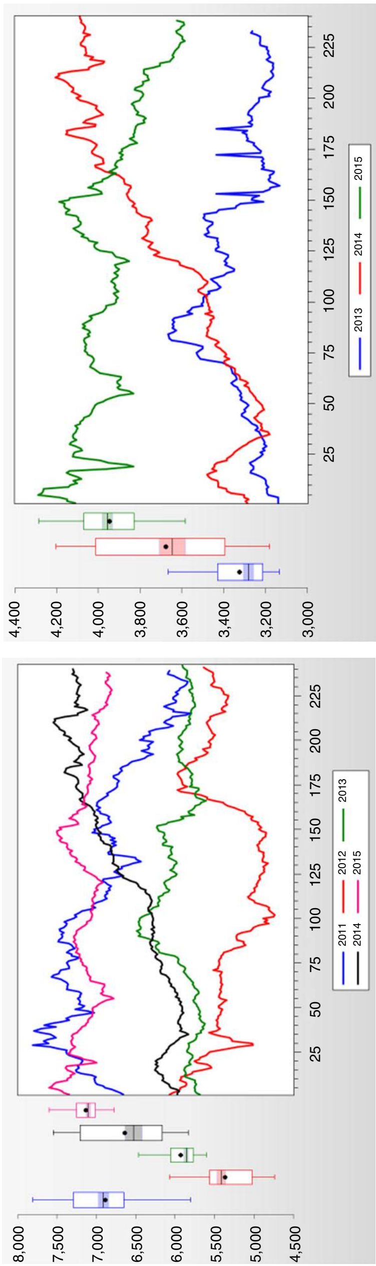
4.1 Case study: CSE

The study was carried out on the basis of secondary data, which were obtained from CSE, Central Bank of Sri Lanka financial reports, different types of background readings, and other relevant sources.

This study mainly focusses on an attempt to understand the trend and cyclic variations as well as to predict future behaviours in CSE, Sri Lanka. Daily trading data of two main price indices, namely, All Share Price Index (ASPI) and S&P Sri Lanka 20 Price Index (SL20), from January 2011 to December 2015 were extracted and tabulated for calculation.

As a developing market, high-volatile fluctuations with unstable patterns are a common phenomenon in CSE. According to the run sequence patterns in ASPI and SL20 in Figure 1, any significant correlation have not been seen between annual data patterns during the past five years between 2011 and 2015. Furthermore, considerable noise with significant trend and observations has been occurring during the sample period. As a result of the disorderly patterns with linear trend behavioural patterns, it is practically difficult to use traditional time series approaches for forecasting and predicting market indices based on long past data.

As a result of these unpredictable irregular patterns, daily trading data during the last two-quarter from September 2015 to December 2015 were extracted and tabulated for our calculations. The data sample consisted of the first 85 per cent observations during the training (in-sample or training sample) and the remaining 15 per cent of were considered to test the generalization capabilities of the proposed models.



Source: CSE database (www.cse.lk/list_by_sector.do)

Figure 1.
ASPI/ SL20 fluctuations yearly

4.2 Data pre-processing and stationary/non-stationary checking

Generally, two types of stationary conditions can be seen in the real-world applications. They are as follows: non-stationary random walk with a drift and trend stationary with intercept. So, recognizing stationary conditions based on their behaviours is significant before doing any further analysis.

According to Table I, intercept (ASPI: 0.3335 > 0.05, SL20: 0.2213 > 0.05) and trend components (ASPI: 0.5525 > 0.05, SL20: 0.3245 > 0.05) of the model are not significant under the 0.05 levels.

As a next step, the corresponding autocorrelations for ASPI and SL20 are calculated in association with the *t*-statistics and Ljung-Box *Q*-statistics and plotted in specific lag intervals as shown in Figure 2. According to the Figure 2, ACF plots of ASPI and SL20 depict that the sample autocorrelation values are strong, positive, and gradually declining with respect to the lag values with 5 per cent level of significance limits.

4.3 Grey system-based hybrid approach for stock market forecasting

In the next stage, grey operational models, namely, GM(1,1), unbiased GM(1,1) (UNBG), NGBM and the new proposed grey hybrid methodology (UNBG_BPNN), are used for forecasting price indices in CSE. Furthermore, three different error accuracy standards were employed to pick out the most suitable model for forecasting with limited number of observations. They are as follows: mean absolute deviation (MAD), mean absolute percentage error (MAPE) and root-mean-square error (RMSE).

Measures of forecasting errors for ASPI and SL20 are shown in Tables II and III, respectively.

In Tables II and III, both training and testing results shows that the UNBG_BPNN model gives the minimum MAPE, MAD, and RMSE model accuracies than others. Furthermore, results indicated that, UNBG_BPNN implies the best simulation result than other models.

Furthermore, the error bar plots in Figure 3 also suggested that the new hybrid UNBG_BPNN model generates high-accuracy predictions than other models.

Based on these results, we suggested that unbiased GM(1,1)-based new hybrid model is better in both model building and *ex post* testing stages in the nonlinear data patterns.

5. Concluding remarks

The time series analysis is an essential methodology which comprises the tools for analysing the time series data to identify the characteristics for making future ad-judgements, especially for decision making in economic and finance.

Variable	Trend	Coefficient	SE	*t*-statistic	Prob.
ASPI	D(ASPI(−1))	−0.779576	0.026127	−29.83802	0.0000
	C	2.553689	2.639609	0.967449	0.3335
	@TREND(1)	−0.001942	0.003269	−0.594233	0.5525
SL20	D(SL20(−1))	−0.34621	0.012421	−9.89765	0.0000
	C	0.97672	1.897562	0.84567	0.2213
	@TREND(1)	−0.000923	0.0032134	−0.23456	0.3245

Table I. ADF test for testing the variables

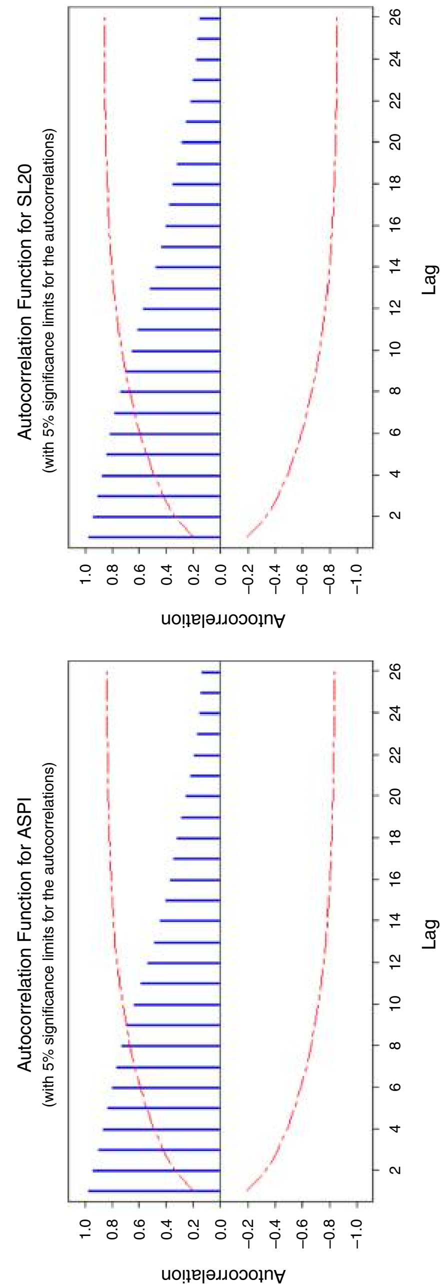


Figure 2.
ACF plot for ASPI/SL20

	ASPI			
Model accuracy	GM(1,1)	UNBG	NGBM	UNBG_BPNN
Training stage				
MAPE	0.46479	0.26970	0.15265	0.03807
MAD	32.328	18.768	10.628	2.65
RMSE	39.03961	21.39443	11.67353	3.169224
Testing stage				
MAPE	0.25835	0.15444	0.073154	0.016025
MAD	18.27062	10.918	5.174	1.134
RMSE	19.41791	12.83028	5.489584	1.185555

Table II.
The ASPI forecasting accuracy

	SL20			
Model accuracy	GM(1,1)	UNBG	NGBM	UNBG_BPNN
Training stage				
MAPE	0.1715	0.180474	0.150177	0.060261
MAD	6.99722	7.38	6.136	2.462
RMSE	9.647412	8.781783	8.330226	4.012608
Testing stage				
MAPE	1.578952	0.75388	0.427885	0.128276
MAD	60.44222	28.862	16.38	4.906
RMSE	62.90244	31.79141	17.74254	5.737841

Table III.
The SL20 forecasting accuracy

Different type of forecasting methodologies can be seen in the literature. These include the bilinear models, the threshold autoregressive, autoregressive conditional heteroscedastic, etc. (Jayathileke and Rathnayake, 2013; Rathnayaka and Seneviratna, 2014; Rathnayaka *et al.*, 2014). However, most of the forecasting mechanisms suggest that linear and nonlinear separate methods are not sufficient to forecast modern financial indices under high volatility. For example, some forecasting models are great at short-term predictions but cannot capture the seasonality or variability with very limited number of sample observations. Furthermore, most of these approaches are more suitable and appropriated only for empirical data studies under the normality, linearity and stationary assumptions.

As a result of these complications with regards to the traditional time series approaches, neural network computing models, grey forecasting mechanisms and new hybrid methodologies have been proposed in the recent literature to handle incomplete, noisy and uncertain data in multidisciplinary systems.

In this study, we developed a new hybrid forecasting methodology based on unbiased GM(1,1) and feed-forward BPNN to handle incomplete, noisy and uncertain data in the daily price indices in CSE, Sri Lanka. The obtained results concluded that, under the chaotic and non-stationary behavioural patterns in the short-term manner (weekly, monthly or quarterly), this new proposed unbiased GM(1,1) hybrid approach is more suitable in model fitting and forecasting with limited sample observations. Furthermore, empirical results indicated more robustness with regards to the possible structure changers in the both model fitting and forecasting with limited data patterns.

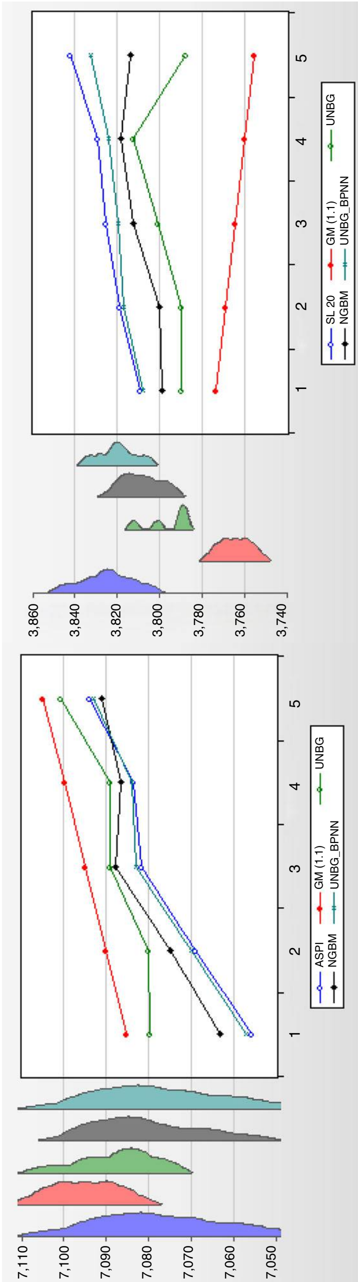


Figure 3.
One week ahead
forecasting for
ASPI/SL20
(testing sample)

However, in the long-term manner with large data samples under the normality, stationary and linearity conditions, the ARIMA and its generalized traditional approaches are still more appropriate.

Finally, we strongly believed that the current study makes significant contributions to researchers and investors toward opening up a new direction to develop new forecasting mechanisms to predict limited data samples under the different real-world conditions.

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