



# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 4 Examination in Engineering: December 2015

Module Number: IS4301

Module Name: Probability and Statistics

[Three Hours]

[Answer all questions, each question carries fourteen marks]

Q1.

a) Explain clearly the following <sup>Two</sup> ~~three~~ sampling methods through at least one example:

- i Simple Random Sampling,
- ii Stratified Sampling.

[4.0 Marks]

b) A researcher is investigating a new method for applying the nickel layer onto the bond pads in the substrate and the thickness of the nickel layer is of particular interest. An assembly with 16 bond pads is examined and the nickel layer thickness is measured for each pad, resulting in the data set shown below.

2.72, 2.79, 2.81, 2.75, 2.77, 2.76, 2.75, 2.75, 2.81, 2.75, 2.74, 2.77, 2.79, 2.78, 2.80, 2.76

- i Find the five-number summary for the nickel layer thickness.
- ii Display the five-number summary in a box-and-whisker-plot.
- iii What does the boxplot tell you about the nickel layer thickness?

[5.0 Marks]

c) A factory has two assembly lines, each of which is shut down (S), operating at partial capacity (P), or at full capacity (F). The sample space and its probabilities are given below.

$$\{(SS), (SP), (SF), (PS), (PP), (PF), (FS), (FP), (FF)\}$$

$$\begin{aligned} P(SS) &= 0.02, & P(SP) &= 0.06, & P(SF) &= 0.05, \\ P(PS) &= 0.07, & P(PP) &= 0.14, & P(PF) &= 0.20, \\ P(FS) &= 0.06, & P(FP) &= 0.21, & P(FF) &= 0.19, \end{aligned}$$

where, (SP) denotes that the first assembly line is shut down and the second one is operating at partial capacity. What is the probability that

- i both assembly lines are shut down?
- ii neither assembly line is shut down?

[2.0 Marks]

- d) A company sells five type of wheelchairs, with type A being 12% of the sales, type B being 34% of the sales, type C being 7% of the sales, type D being 25% of the sales, and type E being 22% of the sales. In addition, 19% of the type A wheelchair sales are motorized, 50% of the type B wheelchair sales are motorized, 4% of the type C wheelchair sales are motorized, 32% of the type D wheelchair sales are motorized, and 76% of the type E wheelchair sales are motorized. If a motorized wheelchair is sold, what is the probability that it is of type C?

[3.0 Marks]

Q2.

- a) A manager supervises the operation of three power plants,  $X$ ,  $Y$ , and  $Z$ . At any given time, each of the three plants can be classified as either generating electricity (1) or being idle (0). With the notation  $(0,1,0)$  used to represent the situation where plant  $Y$  is generating electricity but plants  $X$  and  $Z$  are both idle.

- i Define the sample space for the status of the three plants at a particular point in time.
- ii If the manager's interest is directed only at the number of plants that are generating electricity, then define a random variable and list it all the values.
- iii The probability values for the three power plants are given here.

$$P(0,0,0) = 0.07 \quad P(1,0,0) = 0.16 \quad P(0,0,1) = 0.04 \quad P(1,0,1) = 0.18$$

$$P(0,1,0) = 0.03 \quad P(1,1,0) = 0.21 \quad P(0,1,1) = 0.18 \quad P(1,1,1) = 0.13$$

Find the probability mass function of the random variable.

- iv Find the expected number of power plants generating electricity.

[8.0 Marks]

- b) The discrete random variable  $X$  has a probability distribution given by

$$P(X = x) = x/10 \text{ for } x = 1,2,3,4.$$

Find:

- i  $E(X)$
- ii  $E(X^2)$
- iii  $E(X^2 + 2X - 3)$

[2.0 Marks]

- c) The resistance  $X$  of an electrical component has a probability density function  $f(x) = Ax(130 - x^2)$  for resistance values in the range  $10 \leq x \leq 11$ .

- i Calculate the value of the constant  $A$ .
- ii What is the probability that the electrical component has a resistance between 10.25 and 10.5?

[4.0 Marks]

Q3.

- a) A quality inspector at a glass manufacturing company inspects sheets of glass to check for any slight imperfections. Suppose that the number of these flaws in a glass sheet has a Poisson distribution with parameter  $\lambda$ . The expected number of flaws per sheet is only 0.5.
- Find the probability that there are no flaws in a sheet.
  - Find the probability that there are exactly two flaws in a sheet.
  - Find the probability that there are two or more flaws which are scrapped by the company.

[7.0 Mark]

- b) A company manufactures concrete blocks that are used for construction purposes. Suppose that the weights of the individual concrete blocks are normally distributed with a mean value of  $\mu = 11.0\text{Kg}$  and a standard deviation of  $\sigma = 0.3\text{Kg}$ .

- Find the probability that a concrete block weight less than 10.5Kg and explain your result.
- Suppose that a wall is constructed from 24 concrete blocks. What is the distribution of the total weight of the wall?
- If the average weight of a randomly selected sample of concrete blocks is 10.6Kg, then construct a 95% confidence interval for the population mean weight of the concrete blocks.

[7.0 Mark]

Q4.

- a) Explain the each of the following with an example:

- A Statistic,
- A Parameter.

[2.0 Marks]

- b) Let  $X_1, X_2, \dots, X_n$  be a random sample from the Poisson distribution with the probability mass function

$$f(x|\mu) = \frac{e^{-\mu} \mu^x}{x!}; x = 0, 1, \dots$$

- Use the maximum likelihood estimation method to estimate the parameter  $\mu$ .
- If the following sample represents the number of flaws per glass sheet recorded by a student in his experiment. By assuming that the distribution of the number of flaws per sheet has a Poisson distribution, estimate the mean of the distribution.

0, 1, 1, 1, 0, 0, 0, 2, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 3, 1, 2, 0, 0, 1, 0, 0

[6.0 Marks]

c) Summary statistics for a sample of 60 metal cylinder diameters are given below.

Sample size: 60                      Mean: 49.999                      Standard deviation: 0.134

Maximum: 50.360                      Minimum: 49.740                      Median: 50.010

Construct

i        95%, and

ii       99%,

two-sided confidence intervals for the mean cylinder diameter. Interpret your answers.

[3.0 Marks]

d) A sample of 300 cars having cellular phones and one of 400 cars without phones were tracked for 1 year. The Table Q4 gives summary of the number of these cars involved and not involved in accidents over that year. Test the null hypothesis that having a cellular phone in a car and being involved in an accident are independent. Use the 5% level of significance.

[3.0 Marks]

Q5.

a) Describe each of the following relationships through scatter diagrams.

i        Strong, positive linear relationship

ii       Strong, negative linear relationship

iii      No relationship

[2.0 Marks]

b) A manager of the car plant wishes to investigate how the plant's electricity usage depends upon the plant's production. The data set given in Table Q5 is compiled and provides the plant's production and electrical usage for each month of the previous year. The electrical usage is in units of a million kilowatt-hours, and the production is measured as the value in million-rupees units of the cars produced in that month.

i        Plot the Electricity usage against plant's production.

ii       What is the sample correlation coefficient between the plant's production and Electricity usage? Interpret your answer.

iii      Fit a linear regression model with electricity usage as the response variable and the production as the predictor (explanatory) variable.

iv      If a production level of 5.5 million rupees worth of cars is planned for next month, then predict that the electricity usage.

[12.0 Marks]

**Table Q4: Number of Cars Involved in Accidents over the Year**

	Accident	No Accident
With Cellular Phone	22	278
Without Cellular Phone	26	374

**Table Q5: Plant's Production and Electrical Usage for each Month**

Month	Production (million rupees)	Electricity Usage (million Kwh)
January	4.51	2.48
February	3.58	2.26
March	4.31	2.47
April	5.06	2.77
May	5.64	2.99
June	4.99	3.05
July	5.29	3.18
August	5.83	3.46
September	4.70	3.03
October	5.61	3.26
November	4.90	2.67
December	4.20	2.53