UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 4 Examination in Engineering: December 2015

Module Number: IS4307 Module Name: Probability and Statistics (Old Curriculum)

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

Q1.

a) Explain clearly the following ~~three~~ Two sampling methods through at least one example:

i Simple Random Sampling,

ii Stratified Sampling.

[4.0 Marks]

b) A researcher is investigating a new method for applying the nickel layer onto the bond pads in the substrate and the thickness of the nickel layer is of particular interest. An assembly with 16 bond pads is examined and the nickel layer thickness is measured for each pad, resulting in the data set shown below.

2.72, 2.79, 2.81, 2.75, 2.77, 2.76, 2.75, 2.75, 2.81, 2.75, 2.74, 2.77, 2.79, 2.78, 2.80, 2.76

i Find the five-number summary for the nickel layer thickness.

ii Display the five-number summary in a box-and-whisker-plot.

iii What does the boxplot tell you about the nickel layer thickness?

[5.0 Marks]

c) A company sells five type of wheelchairs, with type A being 12% of the sales, type B being 34% of the sales, type C being 7% of the sales, type D being 25% of the sales, and type E being 22% of the sales. In addition, 19% of the type A wheelchair sales are motorized, 50% of the type B wheelchair sales are motorized, 4% of the type C wheelchair sales are motorized, 32% of the type D wheelchair sales are motorized, and 76% of the type E wheelchair sales are motorized. If a motorized wheelchair is sold, what is the probability that it is of type C?

[3.0 Marks]

Q2.

a) A manager supervises the operation of three power plants, X, Y, and Z. At any given time, each of the three plants can be classified as either generating electricity (1) or being idle (0). With the notation (0,1,0) used to represent the situation where plant Y is generating electricity but plants X and Z are both idle.

i Define the sample space for the status of the three plants at a particular point in time.

ii If the manager's interest is directed only at the number of plants that are generating electricity, then define a random variable and list it all the values.

iii The probability values for the three power plants are given here.

$$P(0,0,0) = 0.07 \quad P(1,0,0) = 0.16 \quad P(0,0,1) = 0.04 \quad P(1,0,1) = 0.18$$
$$P(0,1,0) = 0.03 \quad P(1,1,0) = 0.21 \quad P(0,1,1) = 0.18 \quad P(1,1,1) = 0.13$$

Find the probability mass function of the random variable.

iv Find the expected number of power plants generating electricity.

[8.0 Marks]

b) The resistance X of an electrical component has a probability density function $f(x) = Ax(130 - x^2)$ for resistance values in the range $10 \leq x \leq 11$.

i Calculate the value of the constant A.

ii What is the probability that the electrical component has a resistance between 10.25 and 10.5?

[4.0 Marks]

Q3.

a) A quality inspector at a glass manufacturing company inspects sheets of glass to check for any slight imperfections. Suppose that the number of these flaws in a glass sheet has a Poisson distribution with parameter λ. The expected number of flaws per sheet is only 0.5.

i Find the probability that there are no flows in a sheet and interpret the result.

ii Find the probability that there are two or more flaws which are scrapped by the company.

[7.0 Mark]

b) A company manufactures concrete blocks that are used for construction purposes. Suppose that the weights of the individual concrete blocks are normally distributed with a mean value of $\mu = 11.0Kg$ and a standard deviation of $\sigma = 0.3Kg$. .

i Find the probability that a concrete block weight less than $10.5Kg$ and explain your result.

ii Suppose that a wall is constructed from 24 concrete blocks. What is the distribution of the total weight of the wall?

[5.0 Mark]

Q4.

a) Explain the each of the following with an example:

i A Statistic

ii A Parameter

[2.0 Marks]

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson distribution with probability mass function

$$f(x;\mu) = \frac{e^{-\mu}\mu^x}{x!} \quad ; x = 0, 1, \ldots$$

i Use the maximum likelihood estimation method to estimate the parameter μ.

ii The following sample represents the number of flaws per glass sheet recorded by a student in his experiment. By assuming that the distribution of the number of flaws per sheet has a Poisson distribution, estimate the mean of the distribution.

0, 1, 1, 1, 0, 0, 0, 2, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 3, 1, 2, 0, 0, 1, 0, 0

[6.0 Marks]

c) The following are the weights, in decagrams, of 10 packages of chips distributed by a certain company: 46.4, 46.1, 45:8, 47.0, 46.1, 45.9, 45.2, and 46.0. Find a 95% confidence interval for mean weight of the package of electronic chips distributed by this company. Assume that the weights are normally distributed.

[4.0 Marks]

Q5. A manager of the car plant wishes to investigate how the plant's electricity usage depends upon the plant's production. The data set given in Table Q5 is compiled and provides the plant's production and electrical usage for each month of the previous year. The electrical usage is in units of a million kilowatt-hours, and the production is measured as the value in million-rupees units of the cars produced in that month.

a) Plot the Electricity usage against plant's production.

[2.0 Marks]

b) What is the sample correlation coefficient between the plant's production and Electricity usage? Interpret your answer.

[3.0 Marks]

c) Fit a linear regression model with electricity usage as the response variable and the production as the predictor (explanatory) variable.

[5.0 Marks]

d) If a production level of 5.5 million rupees worth of cars is planned for next month, then predict that the electricity usage.

[2.0 Marks]

Table Q5: Plant's Production and Electrical Usage for each Month

Month	Production (million rupees)	Electricity Usage (million Kwh)
January	4.51	2.48
February	3.58	2.26
March	4.31	2.47
April	5.06	2.77
May	5.64	2.99
June	4.99	3.05
July	5.29	3.18
August	5.83	3.46
September	4.70	3.03
October	5.61	3.26
November	4.90	2.67
December	4.20	2.53