UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: August 2015

Module Number: IS5301 **Module Name: Numerical Methods**

[Three hours]

[Answer **all questions,** each question carries 14 marks]

Write down your answers for **PART-A** and **PART-B** in separate booklets

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PART A

Q1.

a) By considering a practical application, briefly explain the importance of the use of numerical methods for solving scientific and/or engineering problems.

[1.5 Marks]

b) i.) Clearly stating the assumptions, prove the Newton-Raphson formula,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ using Taylor's expansion. [2.0 Marks]

ii.) The circle shown in Figure Q1 has a radius 1, and the longer circular arc joining A and B is twice as long as the chord AB. O is the centre of the circle and OM is perpendicular to AB. If the $A\hat{O}M = \theta$, obtain the function $f(\theta)$,

$$f(\theta) = 2\sin(\theta) + \theta - \pi$$

and find the length of the chord AB, correct to 3 decimal places by using Newton-Raphson method.

[5.0 Marks]

iii.) Discuss the advantages and disadvantages (2 each) of 'Newton-Raphson method' of solving non-linear equations.

[1.5 Marks]

c) The velocity distribution of a fluid near a flat surface is given below.

Distance, x (cm)	0.1	0.3	0.5	0.7	0.9
Velocity, V (cm/s)	0.72	1.81	2.73	3.47	3.98

Where x is the distance from the surface and V is velocity. Using the Newton's forward difference method, obtain the velocity at $x = 0.2$ cm and 0.4 cm.

[4.0 Marks]

Q2.

a) The upward velocity of a rocket is given at three different times below.

Time, t (s)	5	8	12
Velocity, V (m/s)	106.8	177.2	279.2

The velocity data is approximated by a quadratic polynomial as,

$V(t) = a_1 t^2 + a_2 t + a_3 \quad ; \quad 5 \le t \le 12$

The coefficients a_1, a_2 and a_3 for the above expression are given by,

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

Find the values of a_1, a_2 and a_3 using Gauss elimination method. Then, find the velocities of the rocket at t = 6, 7.5 and 10 seconds, by using the quadratic polynomial.

[7.0 Marks]

b) Given the system of equations

$$\begin{aligned} x_1 + 5x_2 + 3x_3 &= 28 \\ 3x_1 + 7x_2 + 13x_3 &= 76 \\ 12x_1 + 3x_2 - 5x_3 &= 1 \end{aligned}$$

with an initial guess of $x_1^{(0)} = 1$, $x_2^{(0)} = 0$ and $x_3^{(0)} = 1$,

i.) find whether the system has a strictly diagonally dominant coefficient matrix.

ii.) if so, solve the system and if not, re-arrange the system and solve it by using Gauss Seidel method.

[7.0 Marks]

Q3.

a) The first level of processing what we see involves detecting edges or positions of transitions from dark to bright or bright to dark points in images. These points usually coincide with boundaries of objects. To model the edges, derivatives of functions such as,

$$f(x) = \begin{cases} 1 - e^{-ax}, & x \ge 0 \\ e^{ax} - 1, & x \le 0 \end{cases}$$ need to be found.

i.) Calculate the functions derivative $f'(x)$ at x = 0.1 for a = 0.12, by using the forward divided difference approximation. Use a step size of h = 0.05.

ii.) Calculate the absolute relative true error.

[7.0 Marks]

b) All electrical components, especially off-the-shelf components do not match their nominal value. Variations in materials and manufacturing as well as operating conditions can affect their value. Suppose a circuit is designed such that it requires a specific component value and to determine the confidence interval of the variation in the component value, a probability density function is needed to be integrated. For an oscillator to have its frequency within 5% of the target of 1 kHz, the likelihood of this happening can then be determined by finding the total area under the normal distribution given by,

$$I = \int_{-2.15}^{2.9} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx.$$

i) Use three-point Gauss quadrature rule to find the frequency.

ii) If the exact value = 0.9824 kHz, find the absolute relative true error.

Table 1: Weighting factors and function arguments used in Gaussian quadrature Formulas

Points	2	3	4
Weighting Factors	$c_1 = 1.0000$ $c_2 = 1.0000$	$c_1 = 0.5555$ $c_2 = 0.8888$ $c_3 = 0.5555$	$c_1 = 0.3478$ $c_2 = 0.6521$ $c_3 = 0.6521$ $c_4 = 0.3478$
Function Arguments	$t_1 = -0.5773$ $t_2 = 0.5773$	$t_1 = -0.7746$ $t_2 = 0.0000$ $t_3 = 0.7746$	$t_1 = -0.8611$ $t_2 = -0.3399$ $t_3 = 0.3399$ $t_4 = 0.8611$

[7.0 Marks]

PART B

Q4.

a) Explain the method of solving initial value problem

$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$

by using 4th order Runge-Kutta method. [2.0 Marks]

b) Intensity of radiation is directly proportional to the amount of remaining radioactive substance and given by the differential equation

$$\frac{dy}{dt} = -ky,$$

where k is a constant.

If initially amount of substance is 200g and $k = 10^{-3}$, use
i.) Euler's method
ii.) Second order Runge-Kutta method
with step size $h = 30$ sec. to determine the amount of substance after one minute.

Compare the two solutions in above i) and ii) with the exact solution. [6.0 Marks]

c) A pendulum of length l making an angle θ with the vertical (Figure Q4), satisfies the nonlinear differential equation

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

where $g = 9.81\,m/s^2$ is the acceleration due to gravity. Initially, θ is $0\,rad.$ and angular velocity, $\omega = d\theta/dt$ is $0.2\,rad/s$. When $l = 80cm$, use Euler's theorem with step size $h = 0.5$ seconds to find θ and ω after one second.

[6.0 Marks]

Q5.

a) By giving an example for each, briefly explain the following.

i.) Partial differential equations (PDE)
ii.) Boundary value problem (BVP)

[2.0 Marks]

b) Classify the following equations as elliptic, parabolic or hyperbolic.

i.) $\frac{\partial u}{\partial t} - 4\frac{\partial^2 u}{\partial t^2} = 0$

ii.) $\frac{\partial^2 f}{\partial x^2} - 2\frac{\partial^2 f}{\partial x \partial t} - 2\frac{\partial^2 f}{\partial t^2} = 0$

iii.) $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

iv.) $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = u \sin t$

[2.0 Marks]

c) List advantages and disadvantages of using the explicit method in solving partial differential equations.

Use a suitable method to solve the heat equation,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad \text{for } 0 \le x \le 1 \text{ and } 0 \le t \le 0.01$$

for the initial conditions,

$$u(x,0) = \sin \pi x$$

and the boundary conditions,

$$u(0,t) = u(1,t) = 0$$

with h = 0.25 and k = 0.0025, where h and k are step sizes along x and t axes respectively.

[10.0 Marks]

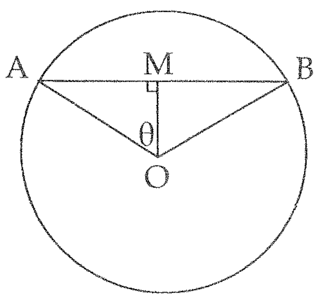

Figure Q1

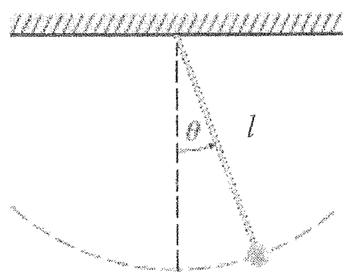

Figure Q4