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UNIVERSITY OF RUHUNA - FACULTY OF ALLIED HEALTH SCIENCES

DEPARTMENT OF PHARMACY

FIRST BPHARM PART I EXAMINATION - FEBRUARY 2023

PH1152: MATHEMATICS SEQ

TIME: TWO HOURS

INSTRUCTIONS

- There are **four** questions in this paper.
- Answer **all** questions.
- Calculators will be provided.
- No paper should be removed from the examination hall.
- Do not use any correction fluid.
- Use illustrations where necessary.

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1. a) Find the following limits:

(i) $\lim_{x\to 1} \dfrac{x^4-1}{x^2-1}$ **[10]**

(ii) $\lim_{u\to 8} \dfrac{u^2-5u-24}{u-8}$ **[10]**

(iii) $\lim_{t\to 1} \dfrac{\sqrt{t}-1}{t-1}$ **[15]**

b) Differentiate $y = x^2 - 4x$ from first principles. **[35]**

c) A certain coal-burning power plant collects air samples of its emissions at various points downwind to determine the concentration of sulfur dioxide at different distances from the plant. The amount of sulfur dioxide measured can be modeled by the function

$$f(x) = \frac{78.35}{x^2},\ x > 0,$$

where x represents the distance downwind in kilometers and $f(x)$ represents the sulfur dioxide concentration in parts per million (ppm).

(i) Determine $f'(x)$. **[10]**

(ii) Evaluate $f(2)$ and $f'(2)$ and interpret each. **[20]**

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2. a) Let $y = \left(\dfrac{x+1}{x-1}\right)^n$, where $n > 0$.

Writing $u = \dfrac{x+1}{x-1}$, find $\dfrac{dy}{dx}$ and hence show that $(x^2-1)\dfrac{dy}{dx} + 2ny = 0$. **[25]**

Continued.

b) The straight line $y = 3x + 1$ is a tangent to the curve given by the function $f(x) = x^2 + ax + b$, where a and b are constants, at the point (0,1). Determine the values of a and b. **[25]**

c) Find the stationary points of the function $f(x) = 4x^3 - 3x^2 - 6x$. **[35]**

Classify the above stationary points as maxima or minima using the second derivative $f''(x)$. **[15]**

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3. a) Consider the two variable function $z = \ln\sqrt{x^2 + y^2}$.

Find the first partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$. **[10]**

Hence show that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 1$. **[05]**

b) Let $f(x,y) = x^7 \ln y + \sin(xy)$. Verify that $\dfrac{\partial^2 f(x,y)}{\partial x \partial y} = \dfrac{\partial^2 f(x,y)}{\partial y \partial x}$. **[30]**

c) Let $f(x,y) = \cos 3x \sin 4y$.

Find the total differential of f at the point $(\pi/12, \pi/6)$. **[25]**

d) Consider the function $g(x,y,z) = 3xz^2 - 2xyz + y^2 z$.

(i) Is the function homogeneous? Justify your answer. **[05]**

(ii) If it is homogeneous, what is the degree of homogeneity? **[05]**

(iii) Show that the function satisfies the Euler's theorem. **[20]**

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4. a) Use the substitution $u = \tan x$ to evaluate the integral $\displaystyle\int \sec^2 x \tan x \, dx$. **[10]**

b) Using the method of integration by parts, evaluate $\displaystyle\int x^2 \ln x \, dx$. **[20]**

c) Show that

$$\int_0^1 \frac{t-1}{t^2 + 3t + 2} \, dt = \ln\left(\frac{27}{32}\right),$$

using the method of partial fractions. **[40]**

d) Test the differential equation $(\cos x - x\cos y)\, dy - (\sin y + y \sin x)\, dx = 0$ for exactness. If it is exact, then find its solution. **[30]**
