

# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 7 Examination in Engineering: August 2015

Module Number: CE7323

Module Name: Computer Analysis of Structures

[Three Hours]

Answer all questions (Each question carries 12 marks)

*All standard notations denote their regular meanings*

- Q1. Analyse the truss structure shown in Fig Q1 using flexibility matrix method. Assume for all members  $AE = 2 \times 10^3$  kN.
- a) Construct the flexibility matrix for structure. [7 Marks]
  - b) Find the displacement at point B. [3 Marks]
  - c) Determine the member force in all members. [2 Marks]
- Q2. Two beams were subjected to a point loading at point B as shown in Fig Q2. Take the lengths of the beam elements  $AB=BC=DB = 10$ m and the parameter  $EI=10 \times 10^6$  Nm<sup>2</sup> for both beams. Using stiffness matrix method of analysis,
- a) Construct the global stiffness matrix of the system. [4 Marks]
  - b) Determine the deformation at the point B. [5 Marks]
  - c) Determine the reaction and the bending moment at the support 'D' for beam DB. [1 Mark]
  - d) Explain briefly, for automatic computation, why stiffness matrix method is more suitable than flexibility matrix method. [2 Marks]

Ignore the axial effect and use the stiffness matrix for a beam element as,

$$[K^e] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

- Q3. Pin-jointed 2D truss is pinned support at Nodes A and C and roller support at Node B as shown in Fig Q3. The Young's modulus  $E = 200$  GPa and cross-section area  $A = 5 \times 10^{-4}$  m<sup>2</sup> for both elements AB and BC. The truss system is subjected to a force 1000 kN at Node B, as shown in Fig Q3.

- a) Find the element stiffness matrix of the 2 elements with respect to a selected global coordinate system. [4 Marks]
- b) Determine the global stiffness matrix of the system. [2 Marks]
- c) Define the boundary condition and loading condition for each node. [2 Marks]
- d) Determine the displacements at Node B. [2 Marks]
- e) Determine support reaction at each node. [2 Marks]

(Use the stiffness matrix for a 2D-bar element as shown below.)

$$[k^e] = \frac{EA}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

where  $c = \text{Cos}\theta$ ,  $s = \text{Sin}\theta$  and  $\theta$  is the anticlockwise angle at node measured from the global X-axis to the local x-axis of the bar element.

- Q4. A beam subjected to different loading is shown in Fig Q4 and is supposed to be analyzed using the finite element technique with three elements. Assume  $EI = 20 \times 10^6 \text{ Nm}^2$ , where E is Young's Modulus and I is the second moment of area of the beam.
- a) Construct the global stiffness matrix of the system. [3 Marks]
  - b) Find the equivalent nodal forces and moments at the four nodes of the beam. [2 Marks]
  - c) Determine the Nodal deformations and reactions. [6 Marks]
  - d) Propose the possible methods to increase the accuracy of the answer in the analysis. [1 Mark]

Ignore the axial effect and use the stiffness matrix for a beam element as shown at the end of Q2.

- Q5. A triangular panel is made of isotropic materials with thickness of 10 mm (shown in Fig Q5). Young Modulus (E) and Poisson's ratio ( $\nu$ ) of the material are 70 GPa and 0.3, respectively. The edge AB is fixed.
- a) Formulate strain-displacement matrix [B]. [6 Marks]
  - b) Construct the stiffness matrix for a plate element. [4 Marks]

c) Determine the nodal deformations at C.

[2 Marks]

Use the stiffness matrix for a 2D-plate element as shown below.

$$[K^n] = \int^v [B]^T [D] [B] d(vol)$$

where

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

and take displacements in the element in x and y direction as  $u(x,y) = a_1 + a_2x + a_3y$   
and  $v(x,y) = a_4 + a_5x + a_6y$ .

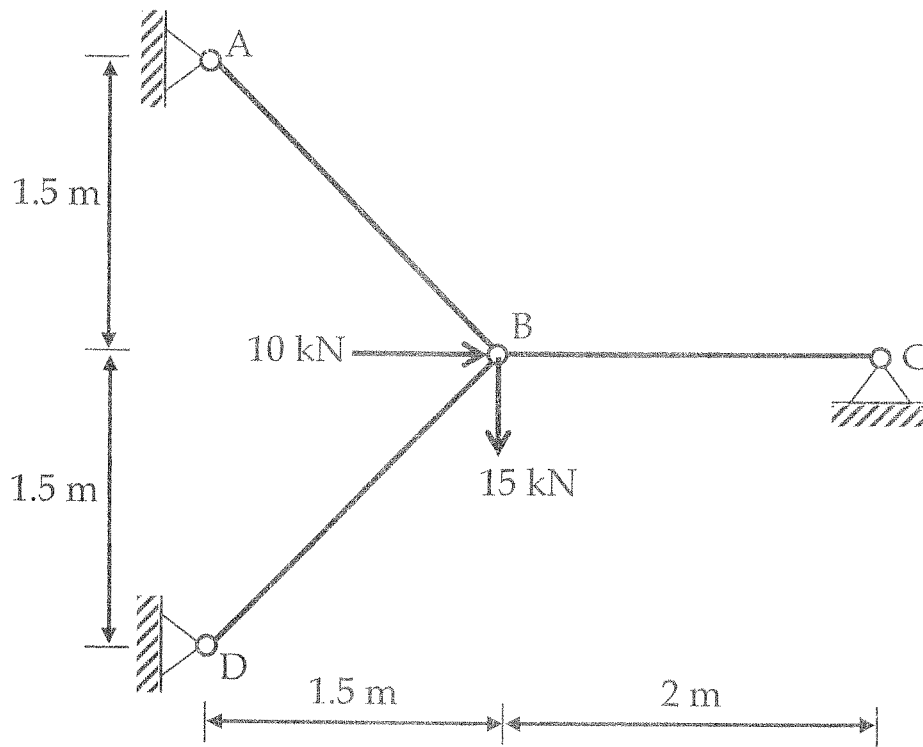


Fig. Q1

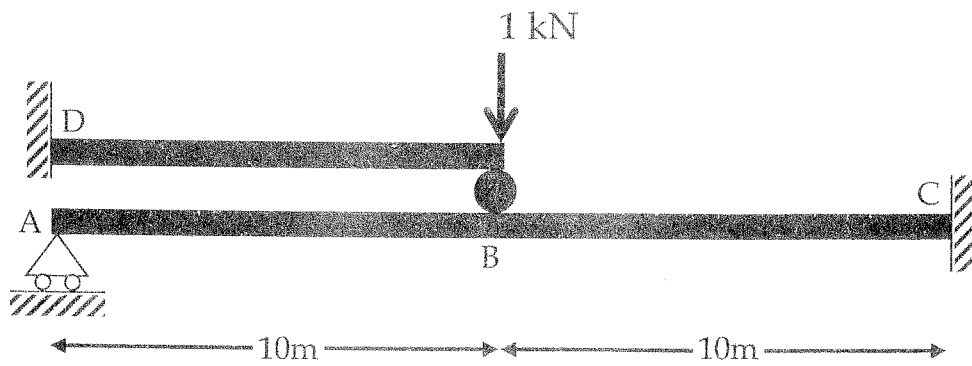


Fig. Q2

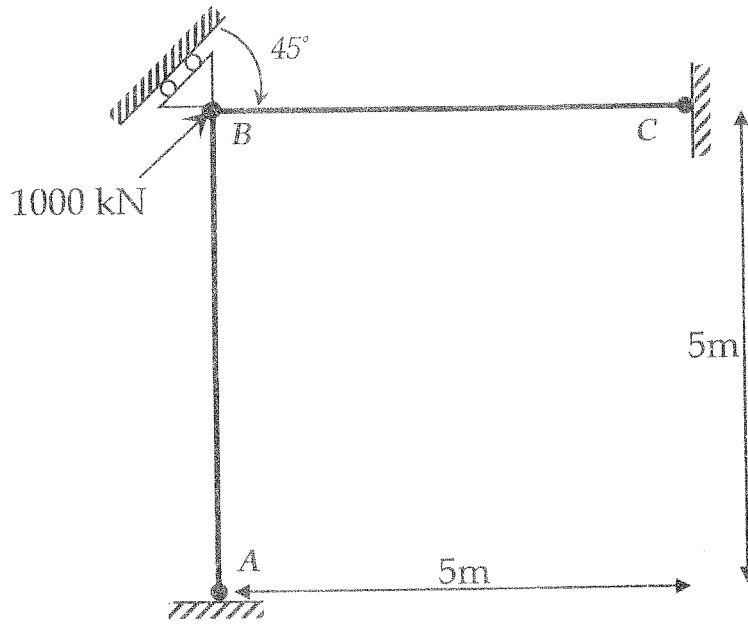


Fig. Q3

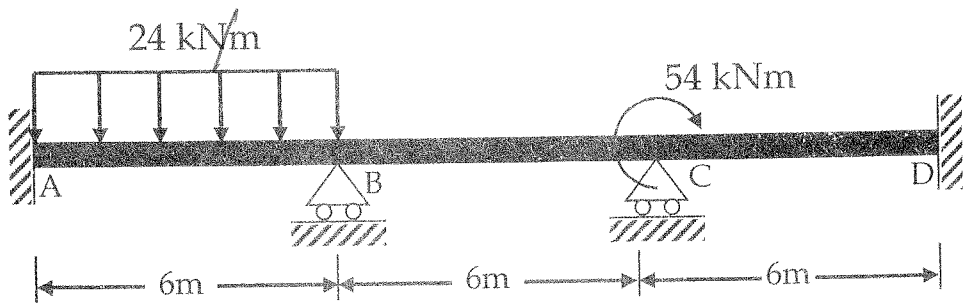


Fig. Q4

