UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: August 2015

Module Number: EE7223 **Module Name: Digital Signal Processing**

[Three Hours]

[Answer **all questions**, each question carries **10 marks**]

Q1 a) Explain explicitly the relationship between the Fourier Transform of a discrete-time sequence (DTFT) and the Discrete Fourier Series (DFS).

[3 Marks]

b) Check whether it is possible to determine the Fourier Series of each of the following discrete-time signals and sketch their magnitude and phase characteristics. If not, explain why.

i) $x[n] = \cos\left(\sqrt{2}\pi n\right)$

ii) $x[n] = \cos\left(\frac{\pi n}{3}\right)$

[7 Marks]

Q2 Consider the real finite-length discrete-time sequence $x[n]$ shown in Figure Q2.

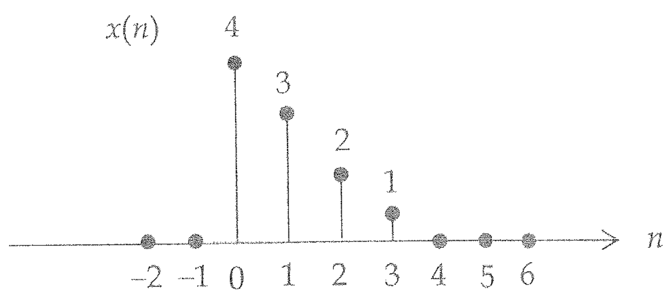

Figure Q2

a) Determine the finite-length sequence $y[n]$ whose 6-point Discrete Fourier Transform (DFT) is

$$Y[k] = e^{-j\frac{2\pi}{6}4k} X[k]$$

where $X[k]$ is the 6-point DFT of $x[n]$.

[4 Marks]

b) Determine the finite-length sequence $w[n]$ whose 6-point DFT is

$$W[k] = \mathrm{Re}\{X[k]\}.$$

[3 Marks]

c) Sketch the finite-length sequence $v[n]$ whose 3-point DFT is

$$V[k] = X[2k], \quad k = 0, 1, 2.$$

[3 Marks]

Q3 a) Compute the 8-point DFT of the discrete-time sequence $x[n] = \left\{\frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0\right\}$.

[3 Marks]

b) Compute the 8-point DFT of the same discrete-time sequence $x[n]$ given in part a) using the Radix-2 decimation-in-frequency FFT (Fast Fourier Transform) algorithm.

Note : Show how you calculate the DFT value at each intermediate node.

[5 Marks]

c) Estimate the computational complexity of the methods in part a) and b) by considering the number of real additions and real multiplications required.

[2 Marks]

Q4 a) What is the difference between an Infinite Impulsive Response (IIR) system and a Finite Impulsive Response (FIR) system?

[2 Marks]

b) Consider the Linear Time-Invariant (LTI) system whose system function is

$$H(Z) = \frac{1 + 2Z^{-1} + Z^{-2}}{1 - \frac{3}{4}Z^{-1} + \frac{1}{8}Z^{-2}}.$$

Determine and draw the signal flow graphs that implement the system function for the following forms.

i) Direct form I structure

ii) Direct form II structure

iii) Parallel form structure using first-order systems

[8 Marks]

Q5 a) Briefly explain the windowing method used in a digital filter design of a FIR discrete-time system.

[3 Marks]

b) Consider a causal continuous-time system with an impulse response $h_c(t)$ and its system function

$$H_c(s) = \frac{s+a}{(s+a)^2 + b^2}.$$

Determine the Z-transform expression of the discrete-time system using the impulse invariance method.

Hint : Let $h[n] = h_c(nT)$ and $e^{-\alpha t}\cos(\beta t)\,u(t) \overset{L}{\leftrightarrow} \dfrac{s+\alpha}{(s+\alpha)^2 + \beta^2}$.

[4 Marks]

c) Suppose that you are asked to design a discrete-time lowpass filter using the bilinear transformation on a continuous-time idea lowpass filter. Assume that the continuous-time prototype has a cut-off frequency $\Omega = 2\pi$ (2000) rad/s and $T = 0.4$ ms. What is the cut-off frequency ω of the resulting discrete-time filter?

Hint : The bilinear transformation is given by, $s = \dfrac{2}{T}\dfrac{\left(1 - Z^{-1}\right)}{\left(1 + Z^{-1}\right)}$.

[3 Marks]