UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7, Examination in Engineering, August 2015

Module Number: EE7256 **Module Name: Scientific Computing**

Part - II

[1 hour and 45 minutes]

[Answer **all questions**, each question carries 7.5 marks]

Q1. a) State the Gauss quadrature rule for integration.

[1 mark]

b) How do you change the integration $\int_a^b f(x)\,\mathrm{d}x$ to the form $\int_{-1}^{1} g(x)\,\mathrm{d}x$?

[1 mark]

c) A company advertises that every roll of toilet paper has at least 250 sheets. The probability that there are 250 or more sheets in the toilet paper is given by

$$P(y \geq 250) = \int_{250}^{\infty} 0.3515\ e^{-0.3881(y-252.2)^2}\,\mathrm{d}y.$$

Approximating the above integral as

$$P(y \geq 250) = \int_{250}^{270} 0.3515\ e^{-0.3881(y-252.2)^2}\,\mathrm{d}y,$$

find the following

i) Given that two point quadrature rule values are given $x_1 = -0.577350269$, $x_2 = 0.577350269$ and $c_1 = 1.0000000$ and $c_2 = 1.0000000$, find the probability using two-point Gauss Quadrature Rule.

[2 marks]

ii) Given that three point quadrature rule values are given $x_1 = -0.774596669$, $x_2 = 0.0000000$, $x_3 = 0.774596669$ and $c_1 = 0.555555556$ and $c_2 = 0.888888889$, $c_3 = 0.555555556$ find the probability using three-point Gauss Quadrature Rule.

[2 marks]

d) Derive formula for multi segment trapezoidal rule for calculating the integral of a function.

[1.5 marks]

Q2. a) Define the following terms

i) Relative true error

ii) Relative approximate error.

[1 marks]

b) The exponential of x, e^x can be calculated by the series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

. We can approximately calculate e^x by truncating the above series to first N as

$$e^x \approx \sum_{n=0}^{N} \frac{x^n}{n!}$$

.

i) Calculate $e^{1.2}$ using first n terms where $N = 1, 2, 3, 4, 5$.

[2 marks]

ii) Calculate approximate error and relative approximate error for $N = 2, 3, 4, 5$.

[2 marks]

iii) Calculate number of significant digits in the solution for $N = 5$.

[1 mark]

c) Explain the round off error using an example.

[0.5 mark]

d) Subtraction of numbers that are nearly equal can create unwanted inaccuracies. Using the formula for error propagation, show that this is true.

[1 mark]

Q3. a) Explain the following

i) Diagonally dominant matrix
ii) Unit vector
iii) Set of linearly independent vectors
iv) Rank of a set of vectors

[2 marks]

b) To infer the surface shape of an object from images taken of a surface from three different directions, one needs to solve the following set of equations.

$$\begin{bmatrix} 0.2425 & 0 & -0.9701 \\ 0 & 0.2425 & -0.9701 \\ -0.2357 & -0.2357 & -0.9428 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 247 \\ 248 \\ 239 \end{bmatrix}$$

The right hand side values are the light intensities from the middle of the images, while the coefficient matrix is dependent on the light source directions with respect to the camera. The unknowns are the incident intensities that will determine the shape of the object. Find the values of $x1$, $x2$, and $x3$ using naive Gauss elimination.

[2 marks]

c) The system of equations

$$25a_1 + 5a_2 + a_3 = 106.8$$
$$64a_1 + 8a_2 + a_3 = 177.2$$
$$144a_1 + 12a_2 + a_3 = 279.2$$

should be solved by using the Gauss-Seidel method. Assume an initial guess of the solution as $a_1 = 1$, $a_2 = 2$ and $a_3 = 5$. Perform 3 iterations and find the relative approximate error.

[2 marks]

d) Factorize the following matrix using LU decomposition method.

$$\begin{bmatrix} 4 & 2 & 1 \\ 6.5 & 2.5 & 1 \\ 9 & 3 & 1 \end{bmatrix}$$

[1.5 marks]

Q4. a) In a uniform rectangular grid find an expressions for

$$\frac{\partial^2 \phi}{\partial x^2} \text{ and } \frac{\partial^2 \phi}{\partial y^2}.$$

[1.5 marks]

b) Heat conduction of domain in Figure Q4.b is governed by the partial differential equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$

The domain is divided into 3×3 uniform grid. Each element is a square with 0.1 units. Find the temperature values in unknown grid points using Finite Difference method.

[3 marks]

c) Electric potential of domain in Figure Q4.c is governed by the partial differential equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$

The domain is divided into 3×3 uniform grid. Each element is a square with 0.1 units. Find the temperature values in unknown grid points using Finite Difference method.

[3 marks]

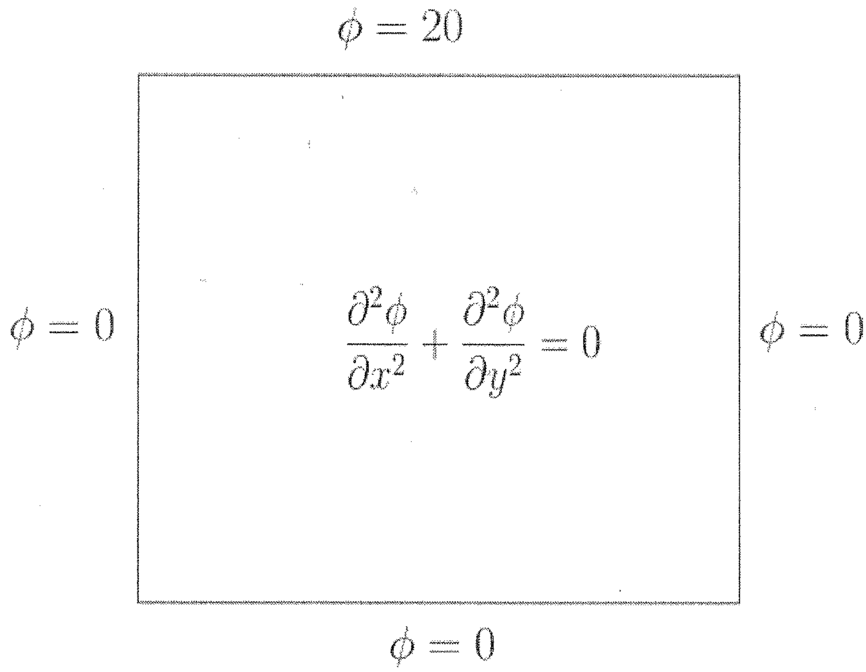

Figure Q4.b: Problem domain for heat distribution

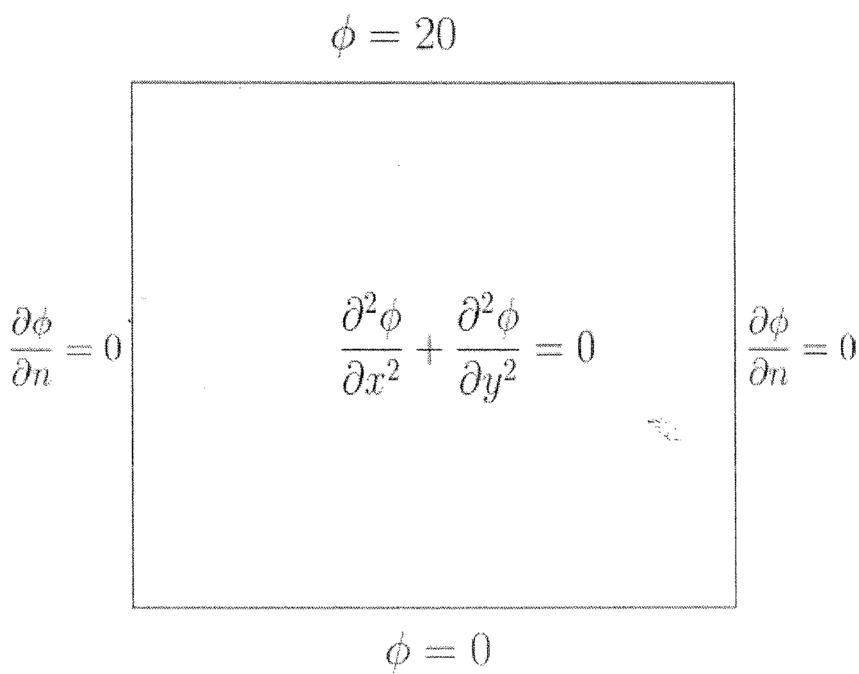

Figure Q4.c: Problem domain for electrical potential distribution