University of Ruhuna-Faculty of Technology

Bachelor of Engineering Technology

Level I (Semester 1) Examination, July 2017

Course Unit: TMS1152 Applied Calculus

Time Allowed 2 hours

Answer all Five(05) questions

All symbols have their usual meaning.

1. (a) Compute following limits.

(i) $\lim_{x\to 0}\left(x^3+2x^2-x+1\right)$

(ii) $\lim_{x\to 1}\dfrac{(1-x)}{(1-\sqrt{x})}$

(b) For the function $f(x)=x^2+2x$ the slope of the tangent line(m_{tan}) at the point $x=1$ can be expressed as follows,

$$m_{tan}=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$$

(i) Compute m_{tan}.

(ii) Write down the equation of the tangent line at $x=1$.

2. For each of the following functions, compute $\frac{dy}{dx}$. (You may use the chain rule if necessary.)

(a) $y=x^2-9x+2$

(b) $y=(x^2+1)(x+2)$

(c) $y=\sqrt{x+\sqrt{x}}$.

(d) $y=u^3-9u$ and $x=3u+2$.

3. (a) If $y=f(x)=x^3-6x^2+4$, find all the points (x,y) on the graph of $f(x)$ where the tangent line is horizontal.

(b) Compute $\frac{dy}{dx}$ of following equations using implicit differentiation.

(i) $3xy+x^2=0$

(ii) $x^2+y^2+e^{(x^2-y^2)}=9$

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4. (a) Prove the following formula is correct by differentiation. First clearly express the corresponding differentiation formula.

$$\int \frac{(x^2+2)}{x}dx = 2\ln(x) + \frac{x^2}{2} + C$$

(b) Compute the following indefinite integral. If necessary use a suitable u-substitution.

(i) $\int (x+1)^2\,dx$

(ii) $\int \frac{1}{x\ln(x)}dx$

(iii) $\int \frac{2x}{x^2+1}dx$

5. (a) Consider the function $y = \sqrt{x}$.

(i) Sketch the graph of the above function.

(ii) Shade the area represented by the definite integral,

$$\int_0^4 \sqrt{x}dx$$

(iii) Compute the area of the above shaded region.

(b) Consider the function $f(x) = (x-2)^2$.

(i) Sketch the graph of the above function.

(ii) Find the slope (m) of the above function at $x = 1$.

(iii) Write down the equation of the tangent line to the above function at $x = 1$.

(iv) Shade the area represented by the definite integral,

$$\int_0^{3/2} (mx + c)dx,$$

where m and c are the slope and the intercept of the tangent line in part (iii) respectively.

(v) Find the area of the shaded region by evaluating the above definite integral.