University of Ruhuna - Faculty of Science
Bachelor of Science General Degree
Level I (Semester I) Examination - December 2020

Subject:Mathematics
Course Unit:MAT111β / MAM1113 (Vector Analysis) **Time**: Two (02) Hours
Answer **ALL Questions**

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1. (a) Let ℓ_1 and ℓ_2 be two non intersecting straight lines in space. Consider the arbitrary points $P_1(x_1,y_1,z_1)$ on ℓ_1 and $P_2(x_2,y_2,z_2)$ on ℓ_2 having position vectors $\underline{r}_1$ and $\underline{r}_2$ respectively with respect to an origin O. Show, apart from sign, that the shortest distance d between the two lines is given by

$$d = \left|\frac{(\underline{r}_2-\underline{r}_1)\cdot(\underline{a}\times\underline{b})}{|\underline{a}\times\underline{b}|}\right|,$$

where $\underline{a} = a_1\underline{i}+a_2\underline{j}+a_3\underline{k}$ and $\underline{b} = b_1\underline{i}+b_2\underline{j}+b_3\underline{k}$ are direction vectors of ℓ_1 and ℓ_2 respectively. **[20]**

Consider the two straight lines

ℓ_1 : passes through the point $(0,9,2)$ with direction vector $(3,-1,1)$
ℓ_2 : passes through the point $(-6,-5,10)$ with direction vector $(-3,2,4)$

The position vectors of P_1 and P_2 are $-3\underline{i}-7\underline{j}+6\underline{k}$ and $3\underline{i}+8\underline{j}+3\underline{k}$ respectively.

(i) Write down the parametric vector equations of ℓ_1 and ℓ_2. **[5+5]**

(ii) Show that the shortest distance d between the two lines is $3\sqrt{30}$. **[20]**

(b) A plane intersects the Ox, Oy and Oz axes, respectively, at $(a,0,0)$, $(0,b,0)$ and $(0,0,c)$, where O is the origin of the coordinate system. Find

(i) a unit normal vector to this plane. **[25]**

(ii) the perpendicular distance between the origin O and this plane. **[05]**

Using the fact that **the scalar triple product of three coplanar vectors is equal to zero,** show that the equation of this plane can be written as

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1.$$

[20]

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2. (a) Find the scalar function $\phi(x,y,z)$ given that $\nabla\phi = 2xyz\underline{i}+(x^2z+1)\underline{j}+x^2y\underline{k}$. **[35]**

(b) State the Green's theorem in the plane. **[10]**

Verify the above theorem for

$$\int_C (y-\sin x)\,dx+\cos x\,dy,$$

where C is the triangle with vertices $(0,0)$, $(\pi/2,0)$, and $(\pi/2,1)$. **[55]**

Continued.

3. State the divergence theorem of Gauss. **[10]**

Verify the divergence theorem for the vector field $\underline{F} = 2x^2y\underline{i} - y^2\underline{j} + 4xz^2\underline{k}$ taken over the closed region D **in the first octant** bounded by $y^2 + z^2 = 9$ and the planes $x = 0, y = 0, z = 0$ and $x = 2$. **[90]**

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4. State the Stokes' theorem. **[10]**

Verify the Stokes' theorem for the vector field $\underline{F} = (x^2 + y^2 - 4y)\underline{i} + 3xy\underline{j} + (2xz + z^2)\underline{k}$ taken over the surface S of the hemisphere $x^2 + y^2 + z^2 = 16$ above the xy-plane. Here, C is the boundary of the hemisphere. **[90]**
