University Of Ruhuna-Faculty of Technology
BACHELOR OF ENGINEERING TECHNOLOGY
Level I (Semester 1) Examination, October 2018

COURSE UNIT: TMS1152 APPLIED CALCULUS

Time Allowed 2 hours

Note : Answer **ALL** Five (05) Questions.

All symbols have their usual meaning.

1. Consider the function,

$$y = \frac{x^2}{x+1}$$

(a) Describe the limits of the above function y as $x \to -1^+$ and $x \to -1^-$.

(b) Find the limit of y as $x \to +\infty$ and $x \to -\infty$.

(c) If dx and dy are differentials, the derivative of the function $y = f(x)$ is given by the quotient $\frac{dy}{dx}$. Apply the differentials dx and dy for above function and find the derivative $\frac{dy}{dx}$ of the above function.

(d) Find at which vaules of x is the above derivative is 0. $\left(\frac{dy}{dx} = 0\right)$.

(e) Using above findings, sketch the shape of the graph of above function.

2. (a) Find the derivative $\left(\frac{dy}{dx}\right)$ of following functions.

   (i) $y = \frac{(2-x)}{(x^2+3x)}$

   (ii) $y = (x^2 + 1)\sec(x)$

(b) Use the properties of logarithm to find the derivative $\left(\frac{dy}{dx}\right)$ of following functions.

(i)

$$y = \frac{(x^2 - 8)^{1/3}\sqrt{x^4 + 2}}{x^6 - 3x + 5}$$

(ii)

$$y = \sqrt[5]{\frac{x - 1}{x^2 - x + 1}}$$

3. Consider the following integral equation.

$$\int \frac{1}{(1 - x^2)^{3/2}} dx = \frac{x}{\sqrt{1 - x^2}} + C$$

(a) What is the corresponding derivative equaton?

(b) Show that the above derivative equation is true by evaluating the derivative.

(c) Suggest a suitable substitution to find the integral $\int \frac{1}{(1-x^2)^{3/2}} dx$.

(d) Evaluate the integral and show that the above integral equation is true.

4. (a) Evaluate the following definite integrals. Use an appropriate substitution if necessary.

(i)

$$\int_0^1 x^2 (1 + x) \, dx$$

(ii)

$$\int_0^3 \frac{e^{\sqrt{3x}}}{\sqrt{x}} dx$$

(b) The area under the function $f(x)$ is denoted by $A(x)$ and has the following relation.

$$\frac{dA(x)}{dx} = f(x)$$

(i) Shade the area under the function $y = x^2$ over the interval $-1 \le x \le 1$.

(ii) Find the area of the above shaded region.

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5. A car passes an electronic toll booth of a highway and traveled to the next exit without stopping. The distance($D(t)$) to the car from the toll booth is measured against the time(t). The time to the exit from the electronic toll booth is 3 hours. The measured distance $D(t)$ (in km) to the car can be well described by the function $D(t) = \frac{-4t^3}{3} + 100t$.

(a) What is the distance between the toll booth and the exit point?

(b) What is the average speed of the car?. (average rate of change of distance with time)

(c) What is the instantaneous speed at $t = 2$ hours?

(d) Sketch the odometer reading of the car against time. (Note:The odometer reading gives the instantaneous speed)

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