University of Ruhuna

Bachelor of Science General Degree
Level I (Semester I) Examination

December 2020

Subject: Mathematics
Course Unit: MAT1142 (Mathematics for Biology)
Time: Two (02) Hours

Answer ALL questions. Calculators will be provided.

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1. a) Consider the complex number $z = 5 + i3$, where i is the imaginary unit.

(i) Write down the complex conjugate, $\bar{z}$ of z

(ii) Show that $z+\bar{z}$ and $z\bar{z}$ are real numbers. **[15 marks]**

b) Find α and β such that $\dfrac{2+i}{1-i} = \alpha + i\beta$, where i is the imaginary unit. **[15 marks]**

c) Using the Binormial theorem, show that

$$\left(m - \frac{3}{n}\right)^4 = m^4 - \frac{12m^3}{n} + \frac{54m^2}{n^2} - \frac{108m}{n^3} + \frac{81}{n^4}.$$ **[30 marks]**

d) (i) Solve the equation :

$$\log_2(x-2) + \log_2 5 = \log_2 x + \log_2 3.$$

(ii) The relation between hydrogen ion concentration, $[H^+]$ and pH value of the solution is given as

$$\text{pH} = -\log_{10}[H^+].$$

Calculate the hydrogen ion concentration, when pH value is 4. **[20 marks]**

e) Prove that

$$\sin 2\theta(\tan\theta + \cot\theta) = 2$$ **[20 marks]**

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2. a) Find $\displaystyle\lim_{x\to 3} \frac{x^2 - 5x + 6}{x - 3}$. **[10 marks]**

b) The population N of the herd of elk is modeled by

$$N(t) = \frac{10(3+4t)}{1+0.1t}.$$

(i) Calculate the size of the herd after 10 years.

(ii) According to the model, determine the limitation size of the herd as time progression. **[20 marks]**

c) Find the first derivative of the following functions:

(i) $y = 2 + 3x + \sin x$

(ii) $y = (x^3 + 5)^3$

(iii) $y = \ln x + e^x$

Continued.

(iv) $y = \frac{3x^2 - 5}{2x+1}$. [40 marks]

d) Find the turning point(s) of the function

$$y = x^3 - 3x.$$

Determine whether the turning point(s) is/are maximum, minimum or point(s) of inflexion using the **second derivative of** y. [30 marks]

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3. a) A three variable function is given by

$$p(x,y,z) = x^3y^2z + xz + x^2y.$$

(i) Find the partial derivatives $\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}$ and $\frac{\partial p}{\partial z}$.

(ii) Show that the total differential of the function p at the point (1,2,3) is given by

$$dp = 43dx + 13dy + 5dz$$

[30 marks]

b) The coefficient of a cubic expression of a gas, α is derived as

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P,$$

where $V, T,$ and P are volume, temperature and pressure respectively.
Calculate the value of α when $P(V - b) = RT$, where b is a constant.

[15 marks]

c) The following data represent the costs (in Rupees) of a sample of 10 postal mailings by a company:

$$30, 62, 45, 78, 36, 62, 76, 51, 98, 42$$

Calculate

(i) mean,
(ii) median,
(iii) mode,
(iv) range,
(v) mean deviation,
(vi) sample variance and
(vii) standard deviation

of the costs of sample. [55 marks]

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4. a) Evaluate the following indefinite integrals:

(i) $\int (3x^2 + \frac{2}{x} - 2)dx$

(ii) $\int \frac{\cos\theta}{2\sin\theta - 1} d\theta$

(iii) $\int (e^{5x} + \cos 3x)dx$ [35 marks]

Continued.

b) It is given that $\int_m^n (2x-1)dx = 4$ and $\int_m^n dx = 1$.
Find m and n. **[20 marks]**

c) Consider the differential equation:
$$(2xy - 3x^2)dx + (x^2 - 2y)dy = 0.$$
Show that the above differential equation is exact and solve it. **[20 marks]**

d) An evergreen nursery usually sells a certain shrub after 6 years of growth and the growth rate during that period is approximated by
$$\frac{dh}{dt} = 1.5t + 5,$$
where t is the time in years and h is the height in centimeters. The seedlings are 12cm tall when planted ($t = 0$).

(i) Find the height of shrubs after t years.

(ii) How tall are the shrubs when they are sold?

[25 marks]
