University of Ruhuna

Bachelor of Science (General) Degree
Level II (Semester I) Examination
January 2022

Subject: Applied Mathematics
Course Unit: AMT 212β (Computational Mathematics) **Time: 02 hours**

Answer All questions.
Allowed to use calculators only supplied by the University.

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1. a) (i) Write down the basic structure of Single Precision IEEE 754 floating point numbers.

(ii) Show that $(-0.125)_{10}$ is $(BE0000)_{16}$ by using single precision IEEE floating point format. [30 **Marks**]

b) (i) Explain the following terms:

i. Absolute error,

ii. Relative error.

(ii) Suppose that result P is obtained by adding of two quantities x and y such that

$$P = x + y.$$

Show that the absolute error of P is

$$e_P \leq e_x + e_y$$

where e_x and e_y are absolute errors in the measurements of x and y respectively.

[30 **Marks**]

c) Students were studying a physical system that gets hotter over time. First, they allowed the system to get hot and measured the temperature at various time t. Their findings were summarized in the following table.

t(sec)	0.5	1.1	1.5	2.1	2.3
T(°C)	32.0	33	34.2	35.1	35.7

(i) Find the linear relationship between time and temperature applying the least square approximation.

(ii) Calculate the temperature of the system at time 1.75 sec. [40 **Marks**]

Note that: The least square estimates for a and b of the least square approximation method $y = ax + b$ can be found by $\hat{b} = \overline{y} - \hat{a}\overline{x}$,

where $\overline{y} = \dfrac{\sum_{i=1}^{n} y_i}{n}$, $\overline{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$ and $\hat{a} = \dfrac{n\left(\sum_{i=1}^{n} x_i y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2}$.

2. a) (i) Show that $f(x) = x^4 + x - 1$ has a root in the interval $[0.5,\ 1.0]$.

(ii) Starting with the interval $[0.5,\ 1.0]$, and use the bisection method twice to the function $f(x) = x^4 + x - 1$, find the interval of width 0.125 which contains the root of $f(x)$.

[30 **Marks**]

b) (i) Explain the graphical interpretation of convergence to the root of the Newton-Raphson method.

(ii) Derive the Newton-Raphson method for finding the q^{th} root of a positive number N, $N^{1/q}$, where $q > 0$.
Assuming initial guess as A or B, show that the square root of $N = AB$ is given by

$$\sqrt{N} \approx \frac{A+B}{4} + \frac{N}{A+B},$$

at the second iteration.

[50 **Marks**]

c) (i) In the usual notation, state the iterative formula of the Secant method for finding the root of the function.

(ii) Write down ta advantage of the Secant method over the Newton-Raphson method.

[20 **Marks**]

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3. a) (i) In the usual notation, write down the boundary conditions of the natural cubic spline for $(n+1)$ points: $x_0, x_1, x_2, \ldots, x_n$ whose function values are $f(x_0), f(x_1), f(x_2), \ldots, f(x_n)$ respectively.

(ii) Find the unknown variables a, b, c, and d of the given natural cubic spline.

$$S(x) = \begin{cases} S_1(x) = 2(x-1) - (x-1)^3 & x \in [1,2] \\ S_2(x) = a + b(x-2) + c(x-2)^2 + d(x-2)^3 & x \in [2,3] \end{cases}$$

[40 **Marks**]

b) A third degree polynomial passes through the points $(0,-1)$, $(1,1)$, $(2,1)$, and $(3,-2)$.

(i) Determine the polynomials using Newton's forward and backward difference interpolation formulas.

(ii) Find the value at 1.5.

[40 **Marks**]

c) Suppose that data points $x_0, x_1, x_2, \ldots, x_n$ be equally spaced such that

$$(x_1 - x_0) = (x_2 - x_1) = \ldots = (x_n - x_{n-1}) = h.$$

In the usual notation, obtain the following relations:

(i) $f[x_0, x_1] = \dfrac{\Delta f(x_0)}{h}$, (ii) $f[x_0, x_1, x_2] = \dfrac{\Delta^2 f(x_0)}{2!h^2}$.

[20 **Marks**]

Continued...

4. a) Using the Lagrange linear interpolating polynomial, obtain the Trapezoidal rule:

$$\int_a^b f(x)dx = \frac{b-a}{2}(f(a) + f(b)).$$

Hence, Deduce the Composite Trapezoidal Rule to find $\int_a^b f(x)dx$, if the interval $[a, b]$ is subdivided into n parts of equal length h.

[30 **Marks**]

b) State the Simpson's 1/3 rule in approximating the integral $\int_a^b f(x)dx$ in the usual notation.

Hence, obtain the expression of the composite Simpson rule to find $\int_a^b f(x)dx$, if the interval $[a, b]$ is subdivided into even n parts of equal length h.

[30 **Marks**]

c) The velocity of a particle which starts from rest is given by the following table.

Time(s)	0	2	4	6	8	10	12
Velocity(ms^{-1})	0	16	29	40	46	51	57

Evaluate the total distance traveled in 12 seconds using

(i) Composite Trapezoidal rule.

(ii) Composite Simson's 1/3 rule.

[40 **Marks**]