UNIVERSITY OF RUHUNA

Faculty of Engineering

Mid-Semester 4 Examination in Engineering (Repeat): October 2015

Module Number: IS4307 **Module Name: Probability and Statistics (O/C)**

[Two Hours]

[Answer **all questions**, each question carries **five marks**]

Q1 a) The following data represent the length of life in years, measured to the nearest tenth, of 30 similar fuel pumps:

2.0	3.0	0.3	3.3	1.3	0.4
0.2	6.0	5.5	6.5	0.2	2.3
1.5	4.0	5.9	1.8	4.7	0.7
4.5	0.3	1.5	0.5	2.5	5.0
1.0	6.0	5.6	6.0	1.2	0.2

i) Construct a relative frequency distribution.

ii) Compute the sample mean, sample range, and sample standard deviation.

[2 Marks]

b) Consider an experiment in which you select a modeled plastic part, such as a connector, and measure its thickness. The possible values for thickness depend on the resolution of the measuring instrument, and they also depend on upper and lower bounds for thickness,

i) Define the sample space.

ii) If it known that all connectors will be between 10 and 11 millimeters thick, then define the sample space.

iii) If the objective of the analysis is to consider only whether a particular part is low, medium, or high for thickness, then define the sample space.

[1.5 Marks]

c) Three machines A, B and C produce respectively 50%, 30% and 20% of the total number of items of a factory. The percentages of defective output of these machines are 3%, 4% and 5%. If an item is selected at random find the probability that the item is defective.

[1.5 Marks]

Q2 a) A company manufactures metal cylinders that are used in the construction of a particular type of engine. These cylinders, which must slide freely within an outer casing, are design to have a diameter of 50mm. The Company discovers however, that the cylinders it manufactures can have a diameter anywhere between 49.5 and 50.5mm.

i) Define appropriate random variable and its values.

ii) Define the probability density function $f(x)$ for the diameter of a metal cylinder. If $f(x)=1.5-6(x-50)^2$, then show that $\int_{-\infty}^{\infty} f(x)dx=1$.

iii) Find the cumulative distribution function of the metal cylinder.

iv) Find the expected diameter of a metal cylinder.

v) Find the variance of a metal cylinder.

[3 Marks]

b) The discrete random variable X has a probability mass function given by

$$P(X=x)=x/10 \text{ for } x=1,2,3,4.$$

Find:

i) $E(X)$

ii) $E(X^2)$

iii) $E(X^2+2X-3)$

iv) Verify that $E(X^2+2X-3)=E(X^2)+2E(X)-3$.

[2 Marks]

Q3 a) Define the moment generating function, $M_X(t)$ for a discrete random variable X.

[1 Mark]

b) Prove that

$$\frac{d^r}{dt^r}M_X(t)\Big|_{t=0}=\mu_r' \text{ , where } \mu_r'=E(X^r).$$

[2 Marks]

c) Find the moment generating function of the binomial random variable X and

then use it to verify that mean $\mu = np$ and variance $\sigma^2 = npq$.

[2 Marks]

Q4 a) Write down the density function of the normal random variable X with mean μ and variance σ^2.

[1 Mark]

b) Suppose that $Z \sim N(0,1)$. Find:

i) $P(Z \leq 1.34)$

ii) $P(Z \geq -0.22)$

iii) $P(-2.19 \leq Z \leq 0.43)$

iv) $P(0.09 \leq Z \leq 1.76)$

[1 Mark]

c) The thicknesses of glass sheets produced by a certain process are normally distributed with a mean of $\mu = 3.00$ mm and a standard deviation of $\sigma = 0.12$ mm.

i) What is the probability that a glass sheet is thicker than 3.2 mm?

ii) What is the probability that a glass sheet is thinner than 2.7 mm?

[2 Marks]

d) A company manufactures concrete blocks that are used for construction purposes. Suppose that the weights of the individual concrete blocks are normally distributed with a mean value of $\mu = 11.0$ Kg and a standard deviation of $\sigma = 0.3$ Kg. Find the probability that a concrete block weights less than 10.5 Kg.

[1 Mark]

CUMULATIVE NORMAL DISTRIBUTION

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

x	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417
$\Phi(x)$	.90	.95	.975	.99	.995	.999	.9995	.99995	.999995
$2[1-\Phi(x)]$	.20	.10	.05	.02	.01	.002	.001	.0001	.00001