UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: March 2022

Module Number: CE 5205 **Module Name: Structural Analysis III**

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

Q1. a) Explain briefly the need of the yield line method for analysis of slabs.

[2 Marks]

b) Express the virtual work equation. Explain briefly all the parameters in the equation.

[2 Marks]

c) Isotropically reinforced hexagonal slab, which is simply supported along four edges, while fixed supported along two edges, as shown in Figure Q1. The side length of the hexagonal slab is L and overall perimeter length of the slab is , therefore, is '$6L$'. The slab carries a uniformly distributed load of intensity 'p' per unit area.

Determine the corresponding collapse load for the yield line pattern given in Figure Q1, assuming the yield moment per unit length of both positive yield lines and negative yield lines as 'm'.

[8 Marks]

Q2. a) Show that the governing equation and the equations for bending moments (with usual notations and sign convention) for rectangular plates are given by

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

where ,

$$D = \frac{Et^3}{12(1-\nu^2)}$$

[6 Marks]

b) A thin rectangular plate of side dimensions 'a' and 'b' with thickness 't' is shown in Figure Q2. The plate is simply supported along all four edges. It is subjected to a vertical downward load of intensity,

$$p(x,y) = P_o sin\frac{\pi x}{a} sin\frac{\pi y}{b}$$

where, P_o is a constant.

i) Assuming a trail solution for displacement, show that the trail solution satisfies the relevant displacement and boundary conditions.

[2 Marks]

ii) Determine an expression for deflection of the plate. Hence, determine an expression for bending moment of the plates.

[4 Marks]

Q3. a) What are the assumptions in deriving the governing equation for a thin circular plate?

[2 Marks]

b) A flat thin circular plate is used as a bottom of cylindrical tank. The plate is subjected to an internal pressure p (r), which varies according to formula, $p(r) = p_o \frac{r}{a}$, as shown in Figure Q3.

i) Show that the shear force per unit length at r distance from center of the plate can be expressed as $Q = p_o \frac{r^2}{3a}$

[2 Marks]

ii) Determine the maximum deflection of the plate.

[4 Marks]

iii) Determine the required thickness of the plate, if allowable stress of the pate materials is 150MPa, and radius of the plate is 2.5 m, internal pressure, p_0, is 2.5 MPa. Assume that E = 200 GPa, and Poisson ratio is 0:3.

[4 Marks]

Governing equation and the equation for the radial moment of circular plate (with usual notations and sign convention) are given by

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\frac{dw}{dr})\right] = \frac{Q}{D} \quad M_r = -D\left(\frac{d^2w}{dr^2} + \frac{v}{r}\frac{dw}{dr}\right) \quad M_t = -D\left(\frac{1}{r}\frac{dw}{dr} + v\frac{d^2w}{dr^2}\right)$$

where

$$D = \frac{Et^3}{12(1-v^2)}$$

Q4. a) "Structural materials are generally far more efficient in an extensional mode rather than in a flexural mode, making the arch and shell are preferable over the beam and plate"

Do you agree with this statement? Justify your answer providing reasons

[2 Marks]

b) Show that the membrane stresses in a spherical shell (*with usual notations and* *sign convention*) are given by

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = P_r \qquad\qquad P_\phi r r_1 - r_1 N_\theta Cos\phi + \frac{\partial(rN\phi)}{\partial\phi} = 0$$

[4 Marks]

c) A spherical lantern shell stiffened by the upper ring and the lower ring is shown in Figure Q4. The shell structure is having radius of R and carrying a vertical line load of 0.5 pR per unit length and a designed live load of p per unit plan area.

Determine membrane stresses caused by above loading.
(Hint: Compute each load case separately and then combine the results)

[6 Marks]

Q5 a) Explain briefly classification of shells based on geometrical developability.

[4 Marks]

b) A planetarium dome may be approximated as an edge-supported truncated cone, as shown in Figure Q5. The load acting on the shell is its self-weight of 2.5 kPa per unit surface area. Assume that the dome is constructed of 12 cm thick concrete having the radii of the parallel circles equal to 40 m at the base and 25 m at the top, respectively. Determine the membrane stresses in the dome.

[8 Marks]

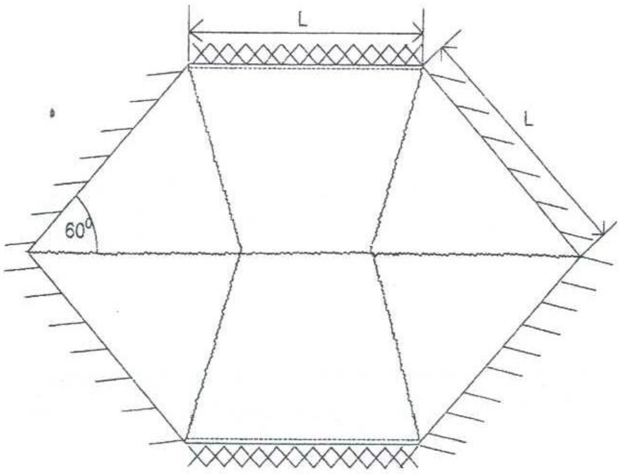

Figure Q1

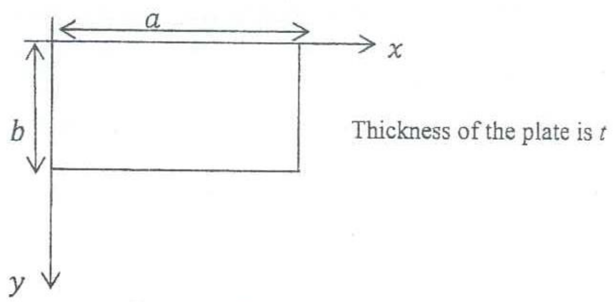

Figure Q2

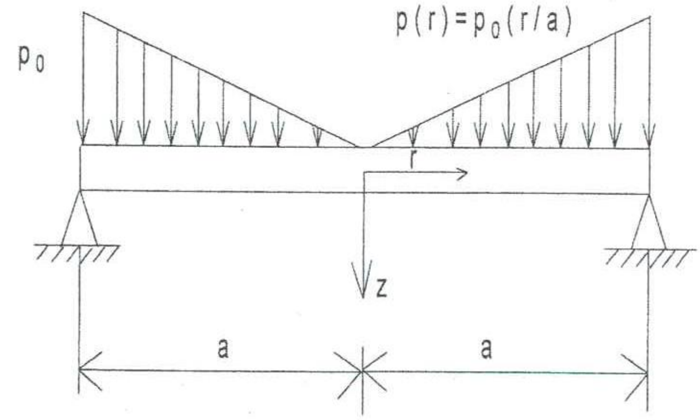

Figure Q3

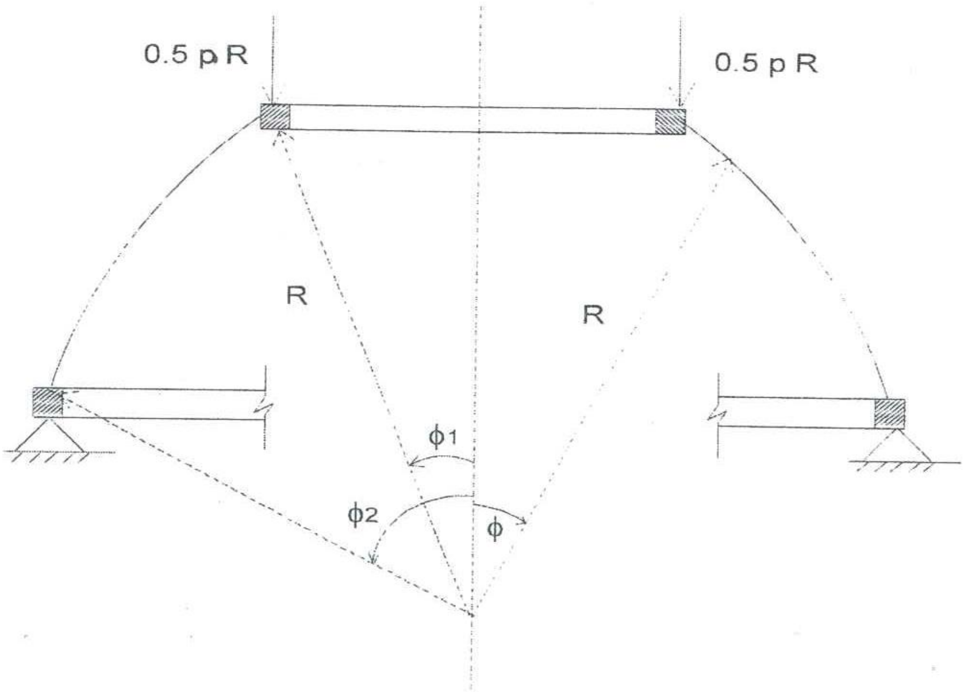

Figure Q4

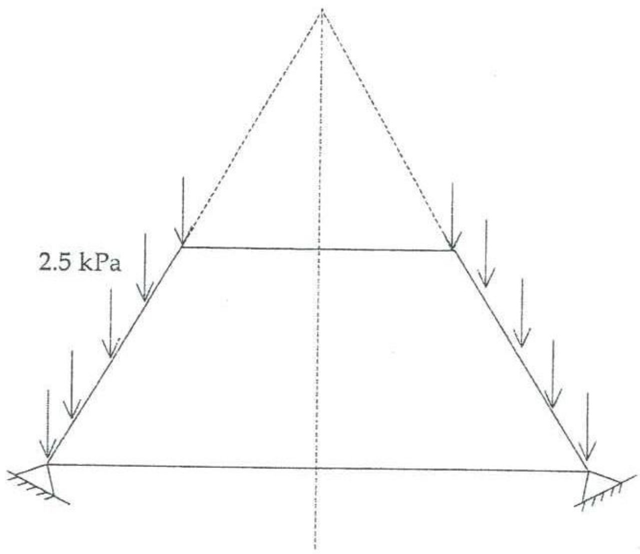

Figure Q5