

University Of Ruhuna-Faculty of Technology  
BACHELOR OF ENGINEERING TECHNOLOGY  
Level I (Semester 1) Examination, November 2019  
COURSE UNIT: TMS1152 APPLIED CALCULUS

Time Allowed 2 hours

Note : Answer ALL Five (05) Questions.

All symbols have their usual meaning.

1. If  $dx$  and  $dy$  are differentials, the DERIVATIVE of the function  $y = f(x)$  is given by the quotient  $\frac{dy}{dx}$ .

(a) Apply the differentials  $dx$  and  $dy$  and find the DERIVATIVE  $\frac{dy}{dx}$  of the following function

$$y = \frac{1}{x^2 + 1}$$

(b) Find the value of the above function at  $x = 1$ .

(c) What is the value of  $\frac{dy}{dx}$  at  $x = 1$ ?

(d) Write the equation of the tangent line to the above graph of  $y = \frac{1}{x^2+1}$  at  $x = 1$ .

2. (a) Find the derivatives ( $\frac{dy}{dx}$ ) of following functions.

(i)  $y = \frac{x^2+1}{2x^3+3}$

(ii)  $y = \sqrt{x^3 + 5}$

(b) Use the properties of logarithm to find the derivative ( $\frac{dy}{dx}$ ) of following functions.

(i)  $y = \frac{xe^{x+1}}{x^2+1}$

(ii)  $y = \sqrt[7]{\frac{x-1}{x^2-x+1}}$

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3. Consider the following integral equation.

$$\int (x + 1)e^x dx = xe^x + C$$

- (a) What is the corresponding derivative equation?
- (b) Show that the above derivative equation is true by evaluating the derivative.
- (c) Use the above integral equation and evaluate the integral,

$$\int x(x^2 + 2)e^{x^2+1} dx$$

(Hint: You may use the substitution  $u = x^2 + 1$ .)

4. Evaluate the following definite integrals. Use an appropriate substitution if necessary.

(a)

$$\int_{-1}^2 4x(1 - x^2) dx$$

(b)

$$\int_0^{\ln(2)} e^{2x} dx$$

(c)

$$\int_0^1 \frac{18x}{1 + 9x^2} dx$$

(d)

$$\int_{-\sqrt{2}}^{-2/\sqrt{3}} \frac{x}{\sqrt{x^2 - 1}} dx$$

5. The area under the function  $f(x)$  is denoted by  $A(x)$  and has the following relation.

$$\frac{dA(x)}{dx} = f(x)$$

- (a) Sketch the shape of the function  $y = x^2 + 1$  on a  $xy$  cartesian coordinate system.
- (b) Shade the area under the function  $y = x^2 + 1$  over the interval  $-1 \leq x \leq 1$ .
- (c) Find the area of the above shaded region.