UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: March 2022

Module Number: IS5306 **Module Name: Numerical Methods**

[Three hours]

[Answer **all questions,** each question carries 12 marks]

Q1. a) The tolerance, *tol,* of the solution in the bisection method is given by

$$tol = \frac{1}{2}(b_n - a_n),$$

where a_n and b_n are the end points of the interval after the n^{th} iteration. The number of interations n that are required for obtaining a solution with a tolerance that is equal to or smaller than a specified tolerance can be determined before the solution is calculated. Show that n is given by

$$n \geq \frac{\log(b - a) - \log(tol)}{\log 2}$$

where a and b are the endpoints of the starting interval.

[2 Marks]

b) The location $\bar{x}$ of the centroid of a segment of a circle is given by:

$$\bar{x} = \frac{2r \sin^3 \alpha}{3\,(\alpha - \sin \alpha \cos \alpha)}$$

By assuming when α is small $\alpha = \sin \alpha$,

i.) Determine the angle α for which $\bar{x} = \frac{3r}{4}$. First, derive the equation that must be solved.

ii.) Find the minimum number of iterations which requires to determine the root. The stopping criteria of this function is 0.01 and the given interval is [0.1, 1.4].

iii.) Use the bisection method to calculate the root of the function.

[6 Marks]

c) Find the solution of the equation:

$$f(x) = x^2 - e^{-0.5x}$$

by using Newton Raphson's method. Calculate only the first three iterations to three significant figures. Use $x = 1$ as the initial guess of the solution.

[4 Marks]

Q2. a) Write down the third derivative approximation equation for the

i.) Newtons Forward difference

ii.) Newtons Backward difference

iii.) Newtons Divided difference

[3 Marks]

b) The geometry of a cam is given in a below figure. A curve needs to be fit through the seven points given in Table 1 to fabricate the cam.

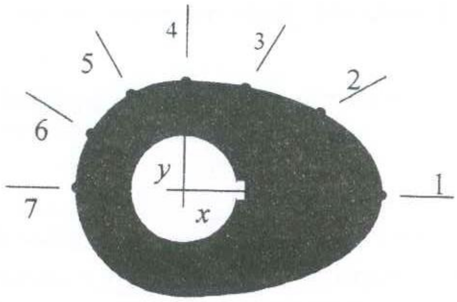

Figure Q2 (b) Schematic of cam profile.

Table 1: Geometry of the cam.

Points	1	2	3	4	5	6	7
$x(cm)$	2.25	1.30	0.65	0.00	-0.60	-1.05	-1.25
$y(cm)$	0.00	0.85	1.15	1.20	1.05	0.65	0.00

i.) If the cam follows a straigt line profile from $x = 1.30$ to $x = 0.65$, what is the value of y at $x = 1.10$ using Newton's divided difference method of interpolation with a first order polynomial.

[2 Marks]

ii.) If the cam follows a biquadratic profile, find the value of y at $x = 1.10$ by using Newton's divided difference method of interpolation with a 4th order polynomial.

[3 Marks]

c) Given the system of equations.

$$x_1 + 5x_2 + 3x_3 = 29$$
$$3x_1 + 7x_2 + 13x_3 = 79$$
$$12x_1 + 3x_2 - 5x_3 = 13$$

with an initial guess of $x_1^{(0)} = 1,\ x_2^{(0)} = 0$ and $x_3^{(0)} = 1$,

i.) determine whether the system has a strictly diagonally dominant coefficient matrix.

[1 Mark]

ii.) if so, solve the system and if not, re-arrange the system and solve it by using Gauss Seidel method.

[3 Marks]

Q3. a) The first level of image processing involves detecting edges or positions of transitions from dark to bright or bright to dark points in images. These points usually coincide with boundaries of objects. To model the edges, derivatives of functions such as,

$$f(x) = \begin{cases} 1 - e^{-ax}, & x \geq 0 \\ e^{ax} - 1, & x \leq 0 \end{cases}$$

need to be found.

i.) Calculate the functions 1st derivative $f'(x)$ at $x = 0.1$ for $a = 0.14$, by using the central difference approximation. Use a step size of $h = 0.05$.

ii.) Calculate the functions 2nd derivative $f''(x)$ at $x = 0.1$ for $a = 0.14$, by using the central difference approximation. Use a step size of $h = 0.05$.

iii.)Calculate the absolute relative true erros (%).

[6 Marks]

b) A Boeing 727-200 airplane of mass $m = 87000\ kg$ lands at a speed of $95\ m/s$ and applies its thrust reversers at $t = 0$. The force F that is applied to the airplane, as it decelerates, is given by $F = -5v^2 - 470000$, where v is the airplane's velocity. Using Newton's second law of motion and flow dynamics, the relationship between the velocity and the position x of the airplane can be written as:

$$mv\frac{dv}{dx} = -5v^2 - 470000$$

where x is the distance measured from the location of the jet at $t = 0$.

Considering five su-bintervals, determine how far the airplane travels before its speed is reduced to $45\ m/s$ by using the composite trapezoidal method to evaluate the integral resulting from the governing differential equation.

[6 Marks]

Q4. a) Briefly explain the following by giving an example for each item.

i.) Ordinary differential equations (ODE)

ii.) Partial differential Equations (PDE)

iii.)Initial value Problem (IVP)

iv.)Boundary value problem (BVP)

[4 Marks]

b) Consider the initial value problem

$$\frac{dy}{dx} = 2x + \cos(x - y) + 3y, \qquad y(2) = 3.$$

Find $y(2.2)$ with step size $h = 0.2$ to four decimal places by using,

i.) Euler method.

ii.) second order Runge-Kutta method.

[4 Marks]

c) Using the following ordinary differential equation with given initial conditions and step size of h, show that the error of 4th order Runge-Kutta method is of order 5 [$O(h^5)$].

$$\frac{dy}{dx} = y, \qquad y(0) = 1$$

[4 Marks]

Q5. a) Classify the following equations as linear or non-linear, and state their order.

i.) $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$

ii.) $\frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} - 6w\frac{\partial w}{\partial x} = 0$

iii.) $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2$

iv.) $2\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial x \partial t} + 3\frac{\partial^2 f}{\partial t^2} + 4\frac{\partial f}{\partial x} + \cos(2t) = 0$

[3 Marks]

b) i) List advantages and disadvantages of using the explicit method in solving partial differential equations.

ii) Use Crank-Nicolson method to solve the partial differential equation,

$$\frac{\partial T}{\partial t} = \frac{1}{3}\frac{\partial^2 T}{\partial x^2}, \quad \text{for } 0 \le x \le 1 \text{ and } 0 \le t \le 1.2$$

with the initial conditions,

$T(x, 0) = 125x$ for $0 \le x \le 0.6$;

$T(x, 0) = 125(1.2 - x)$ for $0.6 < x \le 1$

and the boundary conditions,

$T(0,t) = 0$
$T(1,t) = 25.$

Use, $h = 0.2$ and $k = 0.6$, where h and k are step sizes along x and t axes respectively.

[9 Marks]