UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: March 2022

Module Number: CE7203 (NC) Module Name: Computer Analysis of Structures (NC)

[Three Hour]

[Answer all questions. Marks for each question carries as indicated]

Q1. a) Show that the member flexibility matrix [f] for a beam element with usual notations and clockwise end moments is given by $[f] = \frac{L}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

[3 Marks]

b) Fig.Q1 shows an idealized steel frame, ABCD, used for a steel bridge structure. The frame is simply supported at both ends, A and D. A lateral load is generated at B due to wind action and is acting on the frame as indicated in Fig.Q1. All the elements are equal length, L= 3m and modulus of elasticity, E = 200 GPa, and second moment of area corresponding to in plane bending, I=1.6 x 10^8 mm^4. Using matrix flexibility method, determine following quantities for the frame structure.

i) Member end moments

ii) Horizontal nodal displacement at node B

iii) Support reactions at A and D.

[12 Marks]

Q2. a) Explain briefly how computing time is governed by the matrix stiffness method.

[1 Mark]

b) An idealized frame structure consisting of two members joined together rigidly at Node 2 is supported at Nodes 1 and 3 as shown in Fig. Q2. The frame is guided by a roller support at Node 2. A concentrated force P =100 kN acts horizontally at Node 2. Using matrix stiffness method, perform the followings:

i) Find transformation matrices for each element

[2 Marks]

ii) Determine element stiffness matrix in global coordinates for each element. Use element stiffness matrix for a beam element in local coordinates by ignoring axial effect as:

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

[4 Marks]

iii) Develop structure stiffness matrix using above element stiffness matrices.

[2 Marks]

iv) Determine displacement at Node 2 and reactions at Nodes 1, 2, and 3.

[6 Marks]

Q3. a) i.) List out two advantages of post processing used in finite element analysis.

ii.) Identify the places where it is necessary to place a node, during discretization of a model in finite element analysis.

[2 Marks]

b) Using stiffness equation for 3D continua;

$$\left[K^n\right] = \int^{v} [B]^T [D][B] d(vol)$$

Show that element stiffness for one dimensional bar element is given by

$$\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

[3 Marks]

c) Fig. Q3 shows a one dimensional spring assembly. Element numbers are boxed and node numbers are circled. $k_1 = 300$ kN/m, $k_2 = 200$ kN/m and $k_3 = 500$ kN/m and where k_1, k_2 and k_3 are stiffness of the spring element 1, 2 and 3, respectively.

i.) Assemble the global stiffness matrix of the system of springs.

ii.) Determine the displacement at Node 3.

iii.) Determine support reaction at each node.

[5 Marks]

Q4. Pin-jointed 2D truss is pinned support at Nodes **A** and **C** and roller support at Node **B** as shown in Fig. Q4. The Young's modulus E = 200 GPa and cross-section area A = 4.5×10^{-4} m^2 for both elements AB and BC. The truss system is subjected to a force of 500 kN at Node **B**, as shown in Fig. Q4.

a) Find the element stiffness matrix of the 2 elements with respect to a selected global coordinate system.

[3 Marks]

b) Determine the global stiffness matrix of the system.

[1 Mark]

c) Define the boundary condition and loading condition for each node.

[2 Marks]

d) Determine the displacements at Node **B**.

[2 Marks]

e) Determine the support reactions at each node.

[2 Marks]

(Use the stiffness matrix for a 2D-bar element as shown below.)

$$\left[k^e\right] = \frac{EA}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

where $c = Cos\Theta$, $s = Sin\Theta$ and Θ is the anticlockwise angle at node measured from the global X-axis to the local x-axis of the bar element.

Q5. a) A Beam **ABC** fixed support at Node **A**, roller support at Node **B** and free at Node **C** as shown in Fig Q5(a). The Young's modulus of the beam is E and second moment of area is I and $EI = 3 \times 10^6$ Nm². Consider the load applied at Node **C** as 800 N.

i.) Determine the element stiffness matrix for each element.
ii.) Assemble the global stiffness matrix for the entire system.
iii.) Compute the nodal displacements and rotations.
iv.) Find the reaction forces.

[6 Marks]

b) If the beam is loaded with a clockwise moment of 2500 Nm at Node **B** and a uniformly distributed load of 1200 N/m between **A** and **B** as shown in Fig. Q5(b), compute the nodal displacements and support reactions.

[3 Marks]

c) Propose a possible method to increase the accuracy of the answer in the analysis.

[1 Mark]

(Ignore the axial effect and use the stiffness matrix for a beam element as shown below.)

$$[k^e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

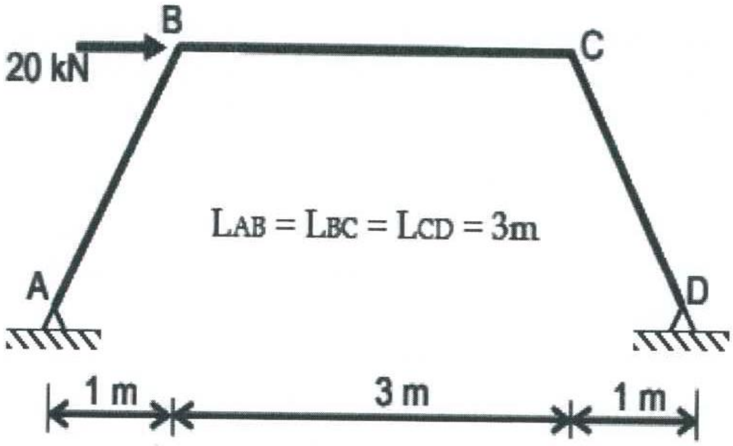

Fig. Q1

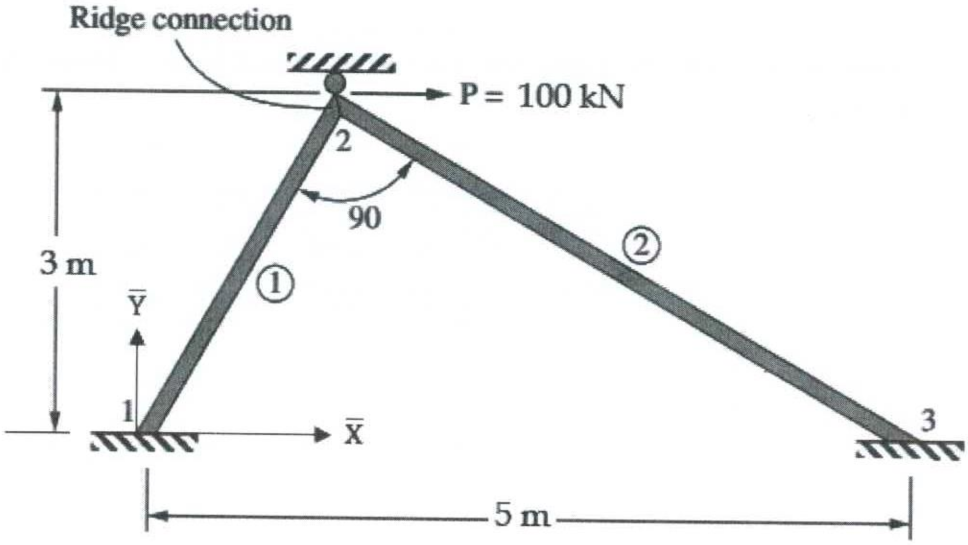

Fig. Q2

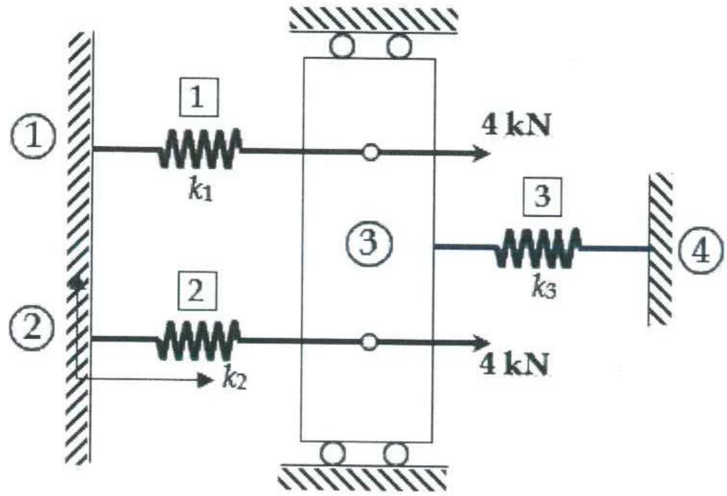

Fig. Q3

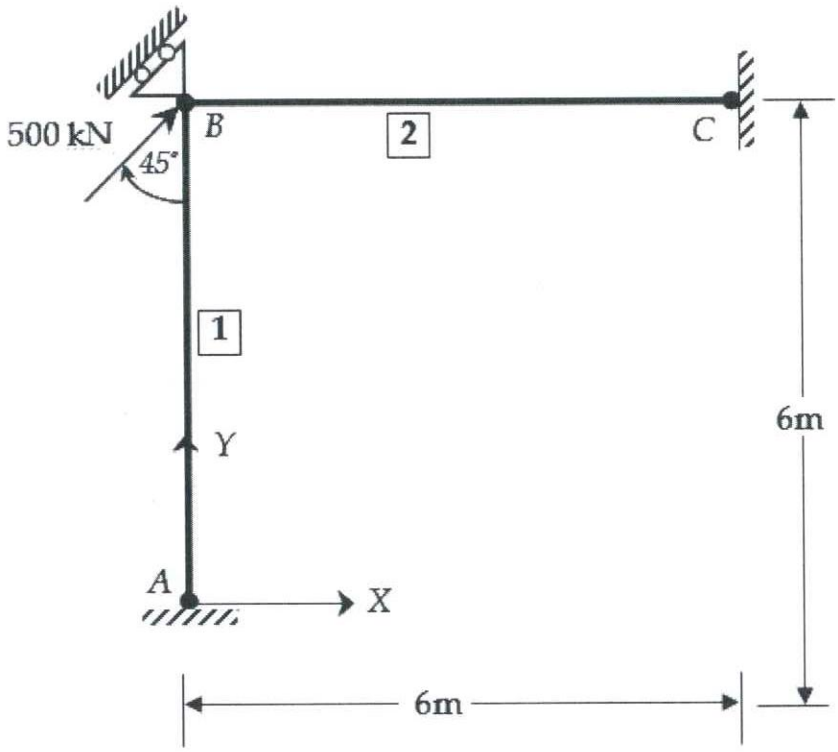

Fig. Q4

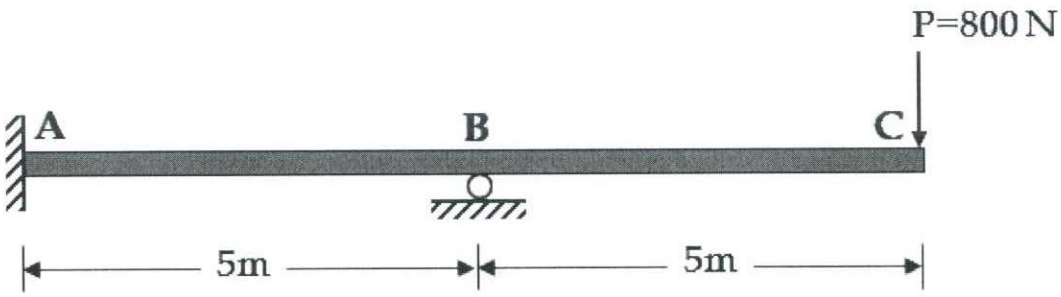

Fig. Q5(a)

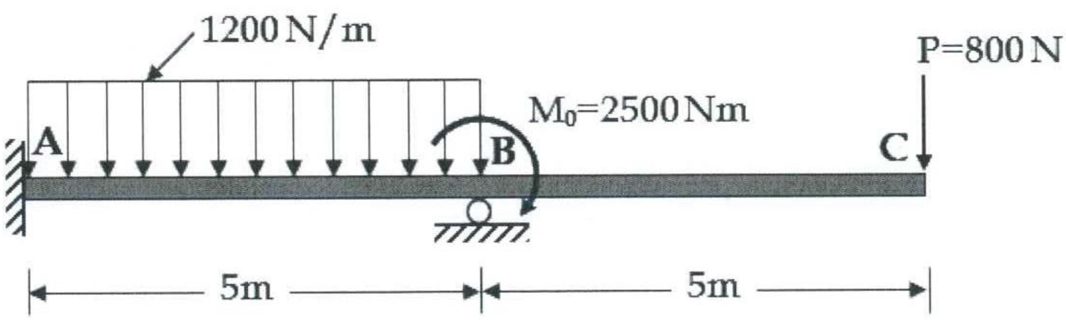

Fig. Q5(b)