UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: March 2022

Module Number: EE 7205 **Module Name: Digital Signal Processing**

[Three Hours]

[Answer all questions, each question carries 10 marks]

All notations and symbols have their usual meaning unless otherwise stated
If necessary, you may use the provided information in Table I and Table II.

Q1 a) The parameters, signal energy and signal power, are used to characterize a signal.

i) What is the difference between an energy signal and a power signal?

ii) Show that the unit step sequence is a power signal.

[3 marks]

b) A system is governed by the difference equation

$$y[n] = 2x[n] - x[n-3].$$

Determine analytically whether the system is

i) linear,

ii) time-invariant.

[3 marks]

c) Refer to $x[n]$ and $y[n]$ signals shown in Figure Q1(a). Note that $y[n]$ is the output of an Linear Time Invariant (LTI) system when $x[n]$ is the input.

i) Find the impulse response $h[n]$ of the system.

ii) What is the response of the system for the input signal $z[n]$ shown in Figure Q1(b)?

[4 marks]

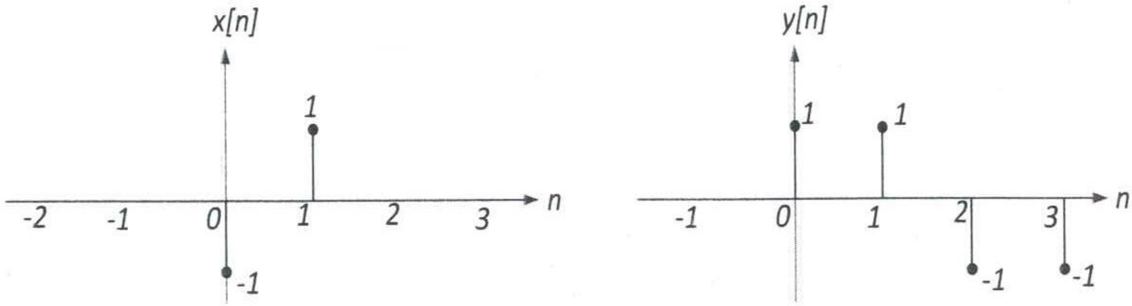

Figure Q1(a)

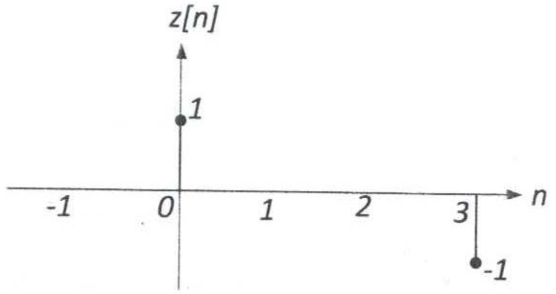

Figure Q1(b)

Q2 a) What is meant by the BIBO stability of a system? Show that the nonlinear system described by the input-output relation

$$y[n] = y^2[n-1] + x[n]$$

is BIBO unstable.

[2 marks]

b) The causal discrete-time LTI system F shown in Figure Q2(a) has input $x[n]$ and output $y[n]$ related by the difference equation $y[n] = 4y[n-1] + x[n]$.

i) Determine the transfer function $F[Z]$ of the system and specify the region of convergence (ROC).

ii) Justify that the system F is unstable.

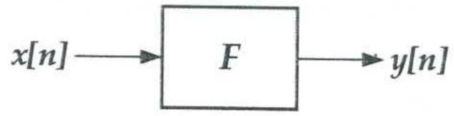

Figure Q2(a)

[2 marks]

c) As shown in Figure Q2(b), a negative feedback is added to the system ***F*** described in part b) to form a new causal LTI system H.

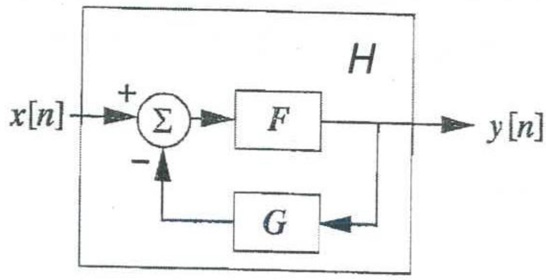

Figure Q2(b)

The impulse response of the LTI system G is given by $g[n] = K\delta[n]$, where $K \in \mathbb{R}$ is a real constant. For what values of the constant K is this overall system H BIBO stable?

[3 marks]

d) $x[n]$ and $y[n]$ are two periodic signals with a period N. Assume $y[n] \neq 0, \forall n$, and

$$z[n] = \sum_{k=0}^{N-1} \frac{x[k]}{y[n-k]}$$

i) Is the signal $z[n]$ periodic? If yes, what is its period?

ii) Let $X[k]$, $W[k]$, $Z[k]$ be the Fourier coefficients of discrete-time sequences $x[n]$, $w[n] = \frac{1}{y[n]}$ and $z[n]$ respectively. Show that

$$Z[k] = N \cdot X[k] \cdot W[k].$$

[3 marks]

Q3 a) The Discrete Fourier Transform (DFT) of the sequence

$$x[n] = \{x[0], x[1], x[2], \dots, x[N-1]\}$$

is given as

$$X[k] = \{X[0], X[1], X[2], \dots, X[N-1]\}.$$

Consider the sequence

$s[n] = \{x[0], x[1], x[2], \dots, x[N-1], x[0], x[1], x[2], \dots, x[N-1]\}$ of length $2N$.

If $S[k]$ is the DFT of the sequence $s[n]$, show that

$$S[k] = \begin{cases} 0; & \text{if } k \text{ is odd} \\ 2X\left[\frac{k}{2}\right]; & \text{if } k \text{ is even} \end{cases}$$

for $k = 0, 1, 2, \dots, (2N-1)$.

[3 marks]

b) $y_R[n]$ is a real-valued sequence with Discrete-time Fourier Transform $Y_R(e^{j\omega})$. The sequences $y_R[n]$ and $y_I[n]$ in Figure Q3(a) are interpreted as the real and the imaginary parts of a complex sequence $y[n]$, i.e., $y[n] = y_R[n] + jy_I[n]$. Determine an expression for $H(e^{j\omega})$ in Figure Q4(a), such that $Y(e^{j\omega})$ is $Y_R(e^{j\omega})$ for negative frequencies and zero for positive frequencies between $-\pi$ and π, i.e.,

$$Y(e^{j\omega}) = \begin{cases} Y_R(e^{j\omega}); & -\pi < \omega < 0 \\ 0; & 0 < \omega < \pi \end{cases}$$

[3 marks]

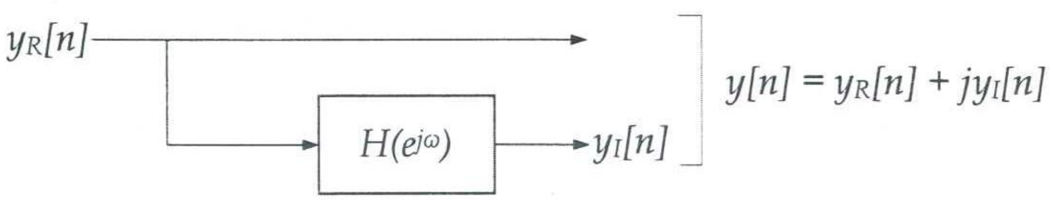

Figure Q3(a)

c) Consider the sequence $x[n] = \{2, 0, 2, 0, 1, 0, 1, 0\}$. Determine the 8-point DFT of $x[n]$ by using the FFT algorithm shown in Figure Q3(b) in **page 7**.

[4 marks]

Q4 a) Sketch the cascade form structure for the realization of the following discrete-time system.

$$H[z] = \left(\frac{1+z^{-1}}{1-\frac{3}{4}z^{-1}}\right)\left(\frac{1+z^{-1}}{1+\frac{1}{2}z^{-1}}\right).$$

[2 marks]

b) A parallel-form realization of an Infinite Impulse Response (IIR) filter can be obtained by performing a partial fraction expansion of the system function $H[Z]$. Determine and sketch the parallel-form structure of the system

$$H[Z] = \frac{1-\frac{1}{2}Z^{-1}}{1-\frac{7}{8}Z^{-1}+\frac{3}{32}Z^{-2}}.$$

[2 marks]

c) Consider the 2-stage lattice filter structure shown in Figure Q4 with lattice coefficients $K_1 = \frac{1}{4}$ and $K_2 = \frac{1}{2}$. Determine the corresponding Finite Impulse Response (FIR) filter coefficients b_1 and b_2 of the direct-form structure obtained by

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2].$$

[3 marks]

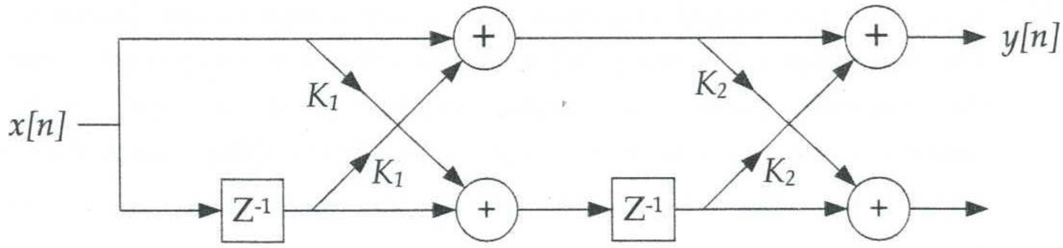

Figure Q4

d) The window method is one of the main techniques used for FIR filter design.
   i) List four windows used for designing FIR filters?
   ii) State an expression for the rectangular window function.
   iii) What are the desirable characteristics of the window function to be used?

[3 marks]

Q5 a) Which filter structure, FIR or IIR has feedback and therefore can be unstable if the coefficients are inappropriately chosen? Briefly discuss your answer.

[1 mark]

b) i) Explain the meaning of phase delay and state an expression for the phase delay of an FIR filter.

ii) Mention the necessary and sufficient condition for linear phase characteristics in FIR filter in terms of phase delay.

[2 marks]

c) A second order digital filter is to be designed from an analog filter

$$H_a(s) = G\frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

having two poles in the s-plane at $p_1 = -1+2j$ and $p_2 = -1-2j$, and two zeros at $z_1 = j$ and $z_2 = -j$ using the bilinear transformation method characterized by the mapping $s = \frac{z-1}{z+1}$.

i) Is the resulting digital filter BIBO stable? Briefly explain your answer.

[2 marks]

ii) Let the frequency response of the resulting digital filter as $H(\omega)$, i.e., the Discrete-time Fourier Transform (DTFT) of its impulse response. Assume that there is only one value of ω for which $H(\omega) = 0$ where $0 < \omega < \pi$. Determine that value of ω.

[1 mark]

iii) Draw the pole-zero diagram of the resulting digital filter.

[1 mark]

iv) If $H(0) = 0.8$, plot the magnitude of the DTFT of the resulting digital filter, $|H(\omega)|$, over $-\pi < \omega < \pi$. Indicate clearly any frequency in which $|H(\omega)| = 0$. Also show that the numerical values of $|H(\omega)|$ for $\omega = \frac{\pi}{2}$ and $\omega = \pi$.

[2 marks]

v) Determine the difference equation of the resulting digital filter.

[1 mark]

Table I: Some common Z-Transform pairs

Signal, $x[n]$	**z-Transform**	**Region of convergence**
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[--n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$(\cos\omega_0 n)u(n)$	$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
$(\sin\omega_0 n)u(n)$	$\frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$

Table II: Frequency analysis of discrete-time signals

Periodic signals		**Aperiodic signals**	
Synthesis equation	$x[n]=\sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$	**Synthesis equation**	$x[n]=\frac{1}{2\pi}\int X(\omega)e^{j\omega n}\,d\omega$
Analysis equation	$c_k=\frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$	**Analysis equation**	$X(\omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$

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Note: Figure Q3(b) should be attached to the answer script.

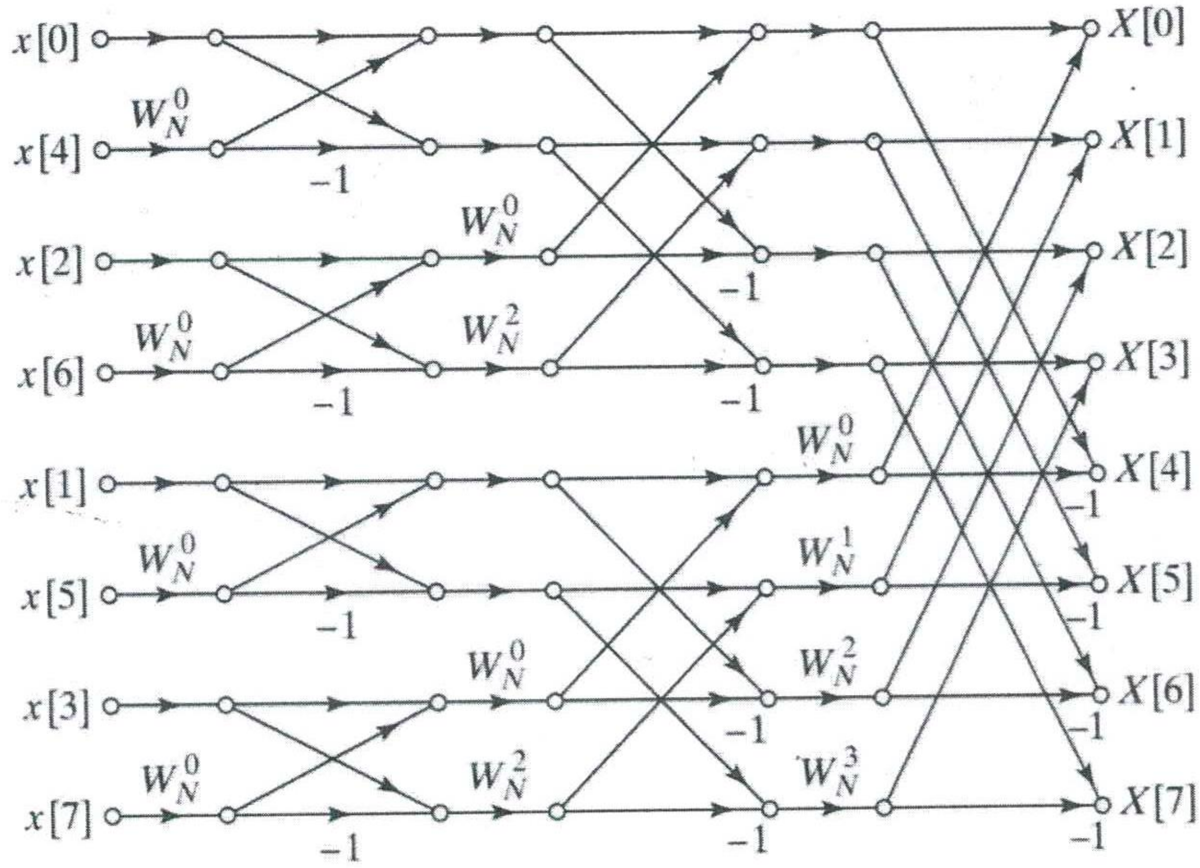

Figure Q3(b)