UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: August 2022

Module Number: ME 3305 Module Name: Modelling and Controlling of Dynamic Systems

[**Three** Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:
Some necessary equations and a partial table of Laplace transformation pairs have been provided on the question paper. **You may make additional assumptions**, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

The standard form of a second-order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;

$T_s = \frac{4}{\zeta\omega_n}$ (±2% settling time);

Percentage Overshoot = $e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

Q1 a) Figure Q1 shows a mass, spring, and damper system with two masses, two dampers and one spring. The mass m1 is subjected to an external force $r(t)$ as shown. The velocity of mass m1 is $v_1(t)$ and the velocity of mass m2 is $v_2(t)$ as shown.

i. Draw the free body diagram and clearly indicate force vectors acting on each mass.

[2 Marks]

ii. Obtain the time domain equations describing <u>velocity</u> of mass m1 and m2.

[4 Marks]

iii. Obtain the Laplace domain model of the system.

[2 Marks]

iv. State any assumptions you made to obtain the above mathematical models.

[1 Mark]

b) A system is described by the following transfer function.

$$\frac{Y(s)}{R(s)} = \frac{15(s+1)}{s^2+9s+14}$$

If this system is subjected to a unit step input, obtain the response $y(t)$ of the system.

[3 Marks]

Q2 a) Figure Q2 shows a passive electrical system where $C = 2\mu F$ and $R1 = R2 = 1M\Omega$, output of the system is $v_2(t)$ and input is $v_1(t)$.

i. Obtain the transfer function of the system in Laplace domain, in terms of R1, R2, and C

[3 Marks]

ii. State the order of the system.

[1 Mark]

iii. Obtain the steady state gain and time constant of the system.

[2 Marks]

iv. State poles and zeros of the system separately.

[1 Mark]

iv. Obtain the unit step response of the system $v_2(t)$ in the time domain.

[2 Marks]

b) Transfer function of a system is given by $G(s) = \frac{a}{s+a}$. The system was excited by a unit step input at the time $t = 0$. Calculate the time t when the system response is 0.39.

[3 Marks]

Q3 a) The RLC system shown in figure Q3 is subjected to a voltage supply $\hat{v}(t) = \hat{V}\cos(\omega t)$. Assume the charge of the capacitor is $q(t)$.

i. State the forcing voltage $\hat{v}(t)$ in complex exponential form.

[1 Mark]

ii. Obtain the mathematical model of the system in time domain in terms of L, R, C, $q(t)$ and $\hat{v}(t)$.

[1 Mark]

iii. Explain why it is possible to use $\hat{q}(t) = \hat{Q}e^{j\omega t}$ with same ω as $\hat{v}(t)$ to replace $\hat{q}(t)$ in (ii) above when the system is in steady state.

[1 Mark]

iv. Obtain an expression to describe complex charge $\hat{q}(t)$.

[3 Marks]

v. The impedance is defined as $\hat{z} = \hat{v}/\hat{\imath}$ where $\hat{v}$ is voltage and $\hat{\imath}$ is current. Obtain the reactance of the system.

[3 Marks]

b) Harmonic oscillation of a system is described as superposition of the following two models.

$\hat{x}_1 = A_1 e^{j(\omega t+\varphi_1)}$, $\hat{x}_2 = A_2 e^{j(\omega t+\varphi_2)}$

i. Draw the phasor diagram to show superposition of two harmonics.

[1 Mark]

ii. Obtain the amplitude A and the phase φ of the vibration of entire system.

[2 Marks]

Q4 The rotational mass, damper, and spring system shown in figure Q4 is subjected to $\tau(t)$ torque. The angle of rotation of the mass is $\theta(t)$.

Notes:

1. Inverse of Matrix A is $A^{-1} = \frac{Adjoint\ (A)}{Determinant(A)}$
2. Transpose of cofactor matrix is defined as adjoint matrix.
3. C_{ij} th element of cofactor matrix is defined as $C_{ij} = (-1)^{(i+j)} \times M_{ij}$ where, M_{ij} is the respective Minor).

4. Minor M_{ij} of 2×2 matrix can be obtained by removing i th row and j th column from 2×2 matrix.

a. Draw the free body diagram, indicating all the forces/torques acting on the mass.

[1 Mark]

b. Obtain the state space model of the system taking θ and $\dot{\theta}$ as states.

[2 Marks]

c. Taking $J = 2\ kgm^2$, $B = 8\ Ns/m$, $K = 6\ N/m$, $\tau(t) = u = 0$ obtain the characteristic equation using the state space model obtained above.

[2 Marks]

d. Obtain the natural frequency and damping ratio of the system.

[2 Marks]

e. Obtain the state transition matrixes $\emptyset(s)$ and $\emptyset(t)$.

[4 Marks]

f. Based on the results of (e), state whether this system is stable or not and give reasons for your answer.

[1 Mark]

Q5 a) A nonlinear dynamics of a system is given as $\dot{x} = F(x) = \sin(x)$.

i. Determine the stability of the system around operating point $x_1 = 0$ rad.

[2 Marks]

ii. Determine the stability of the system around operating point $x_2 = \pi$ rad.

[2 Marks]

iii. Draw the graph showing x vs $F(x)$ and use the results above to indicate the direction of motion of the system when x is between -2π rad to $+2\pi$ rad using arrow heads on the x axis.

[2 Marks]

b) A dynamic system is described by,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u]$$

$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

i. Determine whether this system is controllable.

[2 Marks]

ii. Determine whether this system is observable.

[2 Marks]

c) State advantages of state space system analysis over transfer function model analysis for dynamic systems.

[2 Marks]

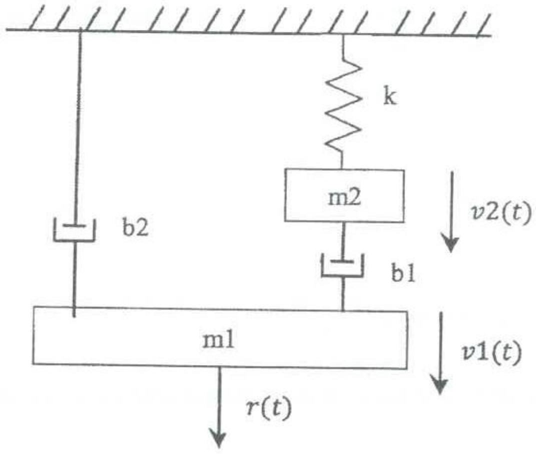

Figure Q1

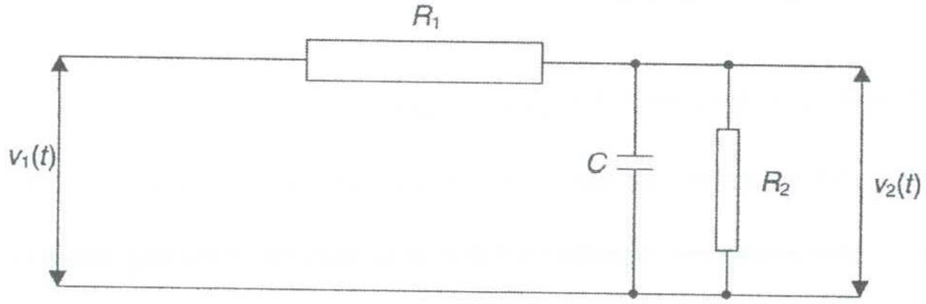

Figure Q2

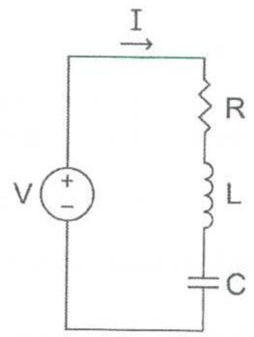

Figure Q3

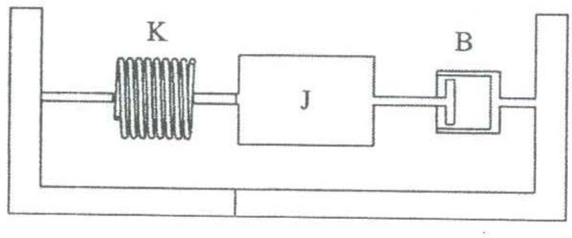

Figure Q4

Laplace Transforms Table

$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2} t^2 e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 + \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 + \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1+at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1, 2, 3$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s} F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t) \quad \textit{Unit impulse}$	$1 \quad \textit{all } s$
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2 f}{df^2}$	$s^2 F(s) - sf(0) - f'(0)$
$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - f^{n-1}(0)$		