UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: July 2022

Module Number: EE3305 **Module Name: Signals and Systems**

[Three Hours]

[Answer **all questions**, each question carries 10 marks]

Q1 a) Consider the block diagrams of two systems shown in Figure Q1.a.

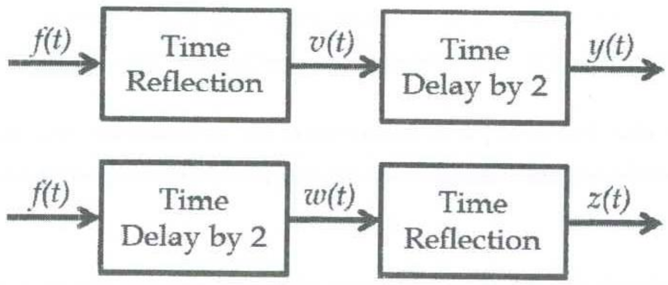

Figure Q1.a

State whether the two systems provide identical outputs for a given input signal $f(t)$. Use the input signal $f(t)$ shown in Figure Q1.b to justify your answer.

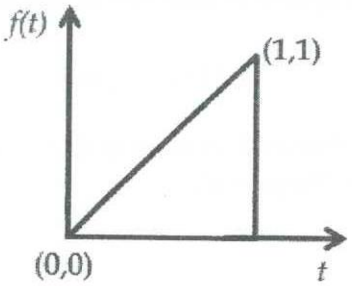

Figure Q1.b

[4 Marks]

b) A continuous-time Linear Time Invariance (LTI) system has the impulse response

$$h(t) = u(t) - u(t-1)$$

i) State a mathematical expression for the system response $y(t)$ when the input signal to the system is $f(t)$.

[3 Marks]

ii) Determine $y(t)$ when $f(t) = u(t)$. Use the graphical convolution method to obtain the answer.

[3 Marks]

Q2 a) i) Determine the exponential Fourier series of a signal $f(t) = B + A\cos(\omega_0 t + \theta)$ where $B > A$. The fundamental frequency of $f(t)$ is ω_0.

[1 Mark]

ii) Use the result obtained in the part a) i) to determine the Fourier series coefficients of a signal $y(t) = B + A\sin(\omega_0 t)$.

[1 Mark]

iii) Plot the magnitude spectrum and phase spectrum of $f(t)$ and $y(t)$.

[2 Marks]

b) Consider a periodic signal

$$f(t) = 0.5 + 4\cos(2\pi t) - 8\cos(4\pi t) \qquad -\infty < t < \infty$$

i) Determine the fundamental frequency of $f(t)$.

[1 Mark]

ii) Determine the Fourier representation for the signal $f(t)$.

[2.5 Marks]

iii) Use the Parsevel's relationship to determine the total average power of the periodic signal using Fourier coefficients obtained in part a) ii).

[2.5 Marks]

Q3 a) Laplace transform of a continuous time signal $f(t)$ is given by $F(s)$.

i) State a mathematical expression for $F(s)$.

[1 Mark]

ii) Show that the Laplace transform of the time shifted signal is

$$\mathcal{L}\,[f(t-t_0)u(t-t_0)] = F(s)\,e^{-st_0}$$

[2 Marks]

iii) Use the result obtained in part a) ii) to determine the Laplace transform of the signal $y(t) = e^{-3t}u(t-4)$.

[2 Marks]

b) Consider the positive feedback system shown in Figure Q3.

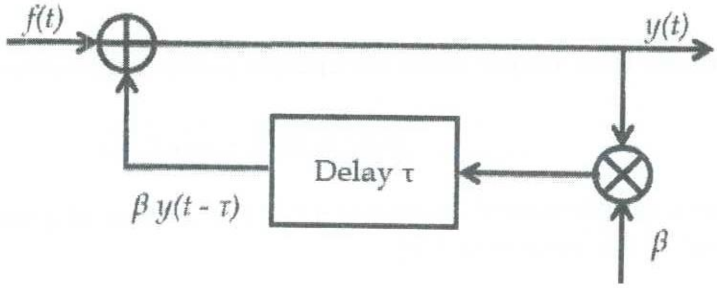

Figure Q3

i) Write a mathematical expression for the system input-output relationship.

[1 Mark]

ii) Determine the system transfer function $H(s)$.

[2 Marks]

iii) Determine the impulse response $h(t)$ of the system. Refer to the unilateral Laplace transform pairs shown in Table 1. Assume $\beta = 1$ and $\tau = 1$.

[2 Marks]

Q4 a) "All practical signals are time-limited. If a signal is time-limited, it cannot be a band-limited signal"

i) Briefly explain the effect of sampling a time limited signal.

[2 Marks]

ii) Explain how the effect given in part a) i) can be minimized.

[2 Marks]

b) Determine the minimum sampling frequency that need to be used to sample the following signals.

i) $f(t) = \sin(200t)$

[3 Marks]

ii) $f(t) = \sin(100t) - 4\cos(100\pi t) + 30\cos(200t)$

[3 Marks]

Q5 a) z-transform of a signal $f[n]$ is given by $F[z]$.

i) State a mathematical expression for $F[z]$.

[1 Mark]

ii) Show that the z-transform of the delayed signal $f[n - n_0]u[n - n_0]$ is

$$z^{-n_0}F[z].$$

[2 Marks]

iii) Using the result obtained in part a) ii), show that the z-transform of the signal $f[n] = u[n] - u[n - 10]$ is

$$\frac{z^{10}-1}{z^9(z-1)}.$$

[2 Marks]

b) Consider the discrete time signal $f[n] = 1 + \cos\left(\frac{\pi n}{2}\right) + \sin(\pi n)$ where $-\infty < n < \infty$.

i) Show that the fundamental frequency ω_0 of $f[n]$ is $\frac{\pi}{2}$.

[2.5 Marks]

ii) Determine the Fourier series of $f[n]$.

[2.5 Marks]

Table 1: A short table of unilateral Laplace transforms

	$f(t)$	$F(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$
6	$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$
7	$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$
8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$
9a	$e^{-at}\cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
9b	$e^{-at}\sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$
10a	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{(r\cos\theta)s+(ar\cos\theta-br\sin\theta)}{s^2+2as+(a^2+b^2)}$
10b	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$