University of Ruhuna-Faculty of Technology

Bachelor of Engineering Technology

Level I (Semester II) Examination, April 2019

Course Unit: TMS1213 Applied Calculus II

Time Allowed 3 hours

Answer all Six(06) questions

All symbols have their usual meaning.

1. (a) Consider the following function,

$$f(x) = x - \frac{1}{x}$$

(i) Find the point x_0 where the above function is discontinous.

(ii) If the conditions for the mean value theorem are satisfied on the following given intervals (a, b), find all the points c satisfying the conclusions of the mean valule theorem such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

A. $(-2, -4)$

B. $(-1, +1)$

C. $(1, 2)$

(b) Compute $\frac{dy}{dx}$ of following equations using implicit differentiation:

(i)
$$x^2 = \frac{x + y}{x - y},$$

(ii)
$$\sin\left(x^2 y^2\right) = x.$$

2. (a) Compute the following integrals using a suitable substitution:

(i) $\int \sin^2 x \cos x dx,$

(ii) $\int \frac{\sin(5/x)}{x^2} dx,$

(iii) $\int \frac{e^{\tan^{-1} x}}{1 + x^2} dx.$

(b) Compute the following integral. Use integration by parts where necessary.

$$\int x^3 e^{x^2} dx$$

3. A solid is formed by revolving the curve $y = x^2 - 1$ around y-axis.

(a) Graph the above function and sketch the shape of the solid object produced on the y interval $[-1, 8]$.

(b) Using the method of slicing express the volume of the solid in terms of a summation of n number of slices.

(c) In the limit of n goes to infinity,where n is the number of slices, above sum will give the exact volume of the object. In this limit write the above sum in definite integral form.

(d) Find the volume of the object by integrating.

4. Arc length(L) of a curve $f(x)$ on the x interval $[a, b]$ can be given by,

$$L = \int_a^b \sqrt{1 + (f'(x))^2 dx}$$

Appy above formular to find the exact arc length of the follwing curves over the given interval.

(a) $y = (x^6 + 8)/(16x^2)$ from $x = 1$ to $x = 2$.

(b) $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 1$.

5. (a) Consider the curves given by, $y = 2\sqrt{2x}$ and $y = x^2$.

(i) Sketch the graph of the above two curves and shade the enclosed region.

(ii) Find the coordinates of the crossing points of above two curves.

(iii) Compute the area of the enclosed region.

(b) The surface of revolution can be generated by revolving a portion of a curve about the x-axis.

(i) A non-negative portion of a curve $y = f(x)$ from $x = a$ to $x = b$ is revolved about the x- axis. Express the surface area of the resuting object as a definite integral.

(ii) The portion of the curve $y = \sqrt{4 - x^2}$ from $x = -1$ to $x = +1$ is revolved about the x-axis.

(iii) Compute the surface area of the resulting object.

6. (a) Consider the followoing differential equation:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$$

(i) What is the order of the above differential equation.

(ii) Show that e^{-2x} and e^{3x} are solutions to the above differential equation.

(iii) A general solution to the above differential equation can be written as,

$$y = c_1 e^{-2x} + c_2 e^{3x}$$

Use following initial conditions to find the unknown constants c_1 and c_2.

$$y(0) = 1$$
$$y'(0) = 8$$

(b) Solve following initial value problems by seperation of variables:

(i) $y - \sec(x)\frac{dy}{dx} = 0$ initial condition: $y(0) = 1$,

(ii) $\frac{dy}{dx} = (1 + y^2)\, 2x$, initial condition: $y(0) = 1$.