UNIVERSITY OF RUHUNA

Faculty of Engineering

Mid-Semester 6 Examination in Engineering: November 2014

Module Number: EE6232 **Module Name: Photonic Devices**

[Two Hours]

Answer all questions, each question carries 5 marks

Q1

a) Coupled-Mode Theory is formulated in terms of coupling of waveguide modes. For an ideal waveguide the normal modes of any optical field at a given frequency ω can be expressed as

$E(r) = \sum A_v E_v(x,y) \exp(i\beta_v z)$

$H(r) = \sum A_v H_v(x,y) \exp(i\beta_v z)$

where E_v and H_v are normalized mode fields propagating in the z direction, A_v is a constant and the summation is over the all discrete indices v.

i. Describe the propagation of the modes when launched in an ideal waveguide?
ii. Can these field expressions be used to formulate coupling between modes?
iii. What is the necessary condition for mode coupling in a waveguide?
iv. What parameter introduces this condition in the formulation of the Coupled Mode Theory?
v. Modify these field expressions so that they can be used to formulate mode coupling.

[0.4 Marks each]

b) The Lorentz Reciprocity Theorem for two arbitrary sets of fields (E_1, H_1) and (E_2, H_2) is given by

$$\nabla \cdot (E_1 \times H_2^* + E_2^* \times H_1) = -i\,\omega\,(E_1 \cdot \Delta P_2^* - E_2 \cdot \Delta P_1)$$

with the usual notation.

Explain how this theorem is used for formulating mode coupling.

[1 Mark]

c) The permittivity ε in a dielectric is generally a 3x3 tensor. Explain the principle of designing an electro-optic device when it is required for ε to be diagonal in the presence of a low frequency electric field.

[2 Marks]

Q2

The forward coupling matrix $F(z{:}z_0)$ for co-directional coupling in a waveguide between two modes A(z) and B(z) propagating in the z direction is given by

$$F(z{:}z_0) = \begin{bmatrix} \dfrac{\{\beta_c \cos\beta_c(z-z_0) - i\delta \sin\beta_c(z-z_0)\}\, e^{i\delta(z-z_0)}}{\beta_c} & \dfrac{ik_{ab} \sin\beta_c(z-z_0)\, e^{i\delta(z+z_0)}}{\beta_c} \\ \dfrac{ik_{ba} \sin\beta_c(z-z_0)\, e^{-i\delta(z+z_0)}}{\beta_c} & \dfrac{\{\beta_c \cos\beta_c(z-z_0) + i\delta \sin\beta_c(z-z_0)\}\, e^{-i\delta(z-z_0)}}{\beta_c} \end{bmatrix}$$

where $\beta_c = (k_{ab}k_{ba} + \delta^2)^{1/2}$. All other notations have their usual meanings.

a) Write the equation for the normal approach to solve forward coupling problems using $F(z{:}z_0)$. [0.5 Marks]

b) For the case where only mode A(z) is launched at z = 0, obtain expressions for the two modes A(z) and B(z) at z. [1 Mark]

c) Obtain expressions for the power in the two modes and the coupling efficiency for a length l. [1.5 Marks]

d) Obtain an expression for the coupling length l_c for the general case and l_c^{PM} for the phase matched case. [1 Mark]

e) Sketch the power exchange between the two modes A(z) and B(z) for phase mismatched and phase matched coupling. [1 Mark]

Q3

a) Figure Q3 shows the structure of a directional coupler.

i. Sketch the refractive index profile for $n_a > n_b$. [0.5 Marks]

ii. Taking the origin of the (x,y) coordinate axes at the center of one of the arms, define the perturbation $\Delta\varepsilon$ and sketch the perturbation for each arm. [2 Marks]

b) The total field in a directional coupler can be shown to be a linear combination of the two independent field patterns of each arm. These are called "supermodes".

i. With reference to Figure Q3, what conditions are necessary to realize a phase matched symmetric dual-channel coupler? [0.5 Marks]

ii. Sketch the supermode fields and the total field for the symmetric dual-channel coupler (assume that the phase matched coupling length = l_c^{PM}). [1 Mark]

iii. Sketch the supermode fields and the total field for an asymmetric dual-channel coupler (assume the coupling length = l_c). [1 Mark]