

University of Ruhuna - Faculty of Technology
Bachelor of Engineering Technology and Information & Communication Technology
Level II (Semester II) Examination, April 2019

Course Unit: TMS2213 Probability and Statistics

Time Allowed Two (02) hours

Answer all Five (05) questions. Calculators are allowed to use for calculations.

1. A computer center has three printers, A, B, and C, which print at different speeds. Programs are routed to the first available printer. The probability that a program is routed to printers are 0.6, 0.3, and 0.1, respectively. Occasionally a printer will jam and destroy a printout. The probability that printers A, B, and C will jam are 0.01, 0.05, and 0.04, respectively. Your program is destroyed when a printer jams. If your program is destroyed what is the probability that printer A is involved? Printer B is involved? Printer C is involved?

2. The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 200 milliliters and a standard deviation of 15 milliliters.
 - (a) What is the probability that a fill volume is less than 190 milliliters?
 - (b) what is the probability that a can contains between 191 and 209 milliliters?
 - (c) how many cans will probably overflow if 230- milliliter cans are used for the next 1000 cans?
 - (d) Determine specifications that are symmetric about the mean that include 99% of all cans.

3.
 - (a) A machine produces metal rods used in an automobile suspension system. A random sample of 15 rods is selected, and the diameters are measured. The resulting data (in millimeters) are as follows:

8.24	8.25	8.20	8.23	8.24
8.21	8.26	8.27	8.20	8.25
8.23	8.21	8.19	8.28	8.24

Assuming that the length of the diameter is normally distributed, answer the following questions:

- (i) Estimate the point estimate of the population mean, μ .
- (ii) Calculate a 95% confidence interval on mean rod diameter.

(b) A company sent seven of its employees to attend a course in building self-confidence. These employees were evaluated for their self-confidence before and after attending this course. The following table gives the scores (on a scale of 1 to 15, 1 being the lowest and 15 being the highest score) of these employees before and after they attended the course.

Before	8	5	4	9	6	9	5
After	10	8	5	11	6	7	9

- (i) Construct a 95% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the score of an employee before attending the course minus the score of the same employee after attending the course.
- (ii) Test at the 1% significance level whether attending this course increases the mean score of employees.

4.

(a) The following table gives information on the number of megapixels and the prices of nine randomly selected point-and-shoot digital cameras that were available on BestBuy.com on January 22, 2019.

Megapixels	10.3	10.2	7.0	9.1	10.0	12.1	8.0	5.0	14.7
Price (\$)	130	150	62	160	200	280	125	60	400

- (i) Find the correlation coefficient.
- (ii) Test at the 1% significance level whether ρ is different from zero.

(b) Direct Mailing Company sells computers and computer parts by mail. The company claims that at least 90% of all orders are mailed within 72 hours after they are received. The quality control department at the company often takes samples to check if this claim is valid. A recently taken sample of 150 orders showed that 129 of them were mailed within 72 hours. Do you think the company's claim is true? Use a 2.5% significance level.

- 5.
- (a) The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.
- (i) What is the probability that for exactly three calls the lines are occupied?
 - (ii) What is the probability that for at least one call the lines are not occupied?
 - (iii) What is the expected number of calls in which the lines are all occupied?
- (b) A company operates four machines in three shifts each day. From production records, the following data on the number of breakdowns are collected:

	Machines			
Shift	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
1	41	20	12	16
2	31	11	9	14
3	15	17	16	10

Test the hypothesis that breakdowns are independent of the shift at 5% level of significance.

Supplementary Statistical Formulas

- Mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \qquad \mu = \frac{1}{N} \sum_{i=1}^N x_i$$

- Variance

$$s^2 = \frac{1}{n-1} \left[\sum_{i=1}^n (x_i - \bar{x})^2 \right] = \frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right] \qquad \sigma^2 = \frac{1}{N} \left[\sum_{i=1}^N (x_i - \mu)^2 \right] = \frac{1}{N} \left[\sum_{i=1}^N x_i^2 - N\mu^2 \right]$$

- For event A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Standardizing a normal random variable ($X \sim N(\mu, \sigma^2)$)

$$z = \frac{X - \mu}{\sigma}$$

- If X_1, \dots, X_n are independent and identically distributed random variables and $X_i \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \sigma^2/n)$, where $\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$

- Tests for a hypothesized population mean (μ_0). Under H_0

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0, 1) \qquad \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

- $(1 - \alpha)100\%$ Confidence interval for the population mean

$$\left(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right) \qquad \left(\bar{x} - t_{n-1, \frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \bar{x} + t_{n-1, \frac{\alpha}{2}} \frac{s}{\sqrt{n}} \right)$$

- The sample size required to estimate μ with a $(1 - \alpha)100\%$ confidence interval:

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{B} \right]^2$$

- Testing for a difference in population means for two independent populations, under H_0 ,

$$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$$

$$\frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{1/n_1 + 1/n_2}} \sim t_{n_1+n_2-2} \quad \text{where } s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t_\nu \quad \text{where } \nu = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

- Testing for a difference in population means for two dependent populations. Under H_0 ,

$$\frac{\bar{d}}{s_d/\sqrt{n}} \sim t_{n-1}$$

where \bar{d} - Mean of paired differences, s_d - Standard deviation of paired differences

- Correlation
Pearson's correlation coefficient (r) between X and Y

$$r = \frac{\sum_i x_i y_i - n\bar{x}\bar{y}}{\sqrt{(\sum_i x_i^2 - n\bar{x}^2)(\sum_i y_i^2 - n\bar{y}^2)}}$$

Test the significance of the correlation coefficient (ρ). Under H_0 ,

$$\frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2}$$

- Regression
Estimated regression coefficients
Slope

$$\hat{\beta}_1 = b_1 = \frac{\sum_i x_i y_i - n\bar{x}\bar{y}}{\sum_i x_i^2 - n\bar{x}^2}$$

Intercept

$$\hat{\beta}_0 = b_0 = \bar{y} - b_1\bar{x}$$

Residual (e_i) = $y_i - \hat{y}_i$

Sum of Squares Regression (SSR) = $\sum_i (\hat{y}_i - \bar{y})^2$ df=1

Sum of Squares Error (SSE) = $\sum_i (y_i - \hat{y}_i)^2$ df=n-2

Sum of Squares Total (SST) = $\sum_i (y_i - \bar{y})^2$ df=n-1

SST=SSR+SSE $F = MSR/MSE \sim F_{1,n-2}$ $R^2 = \frac{SSR}{SST}$

- Pearson χ^2 Test Statistic:

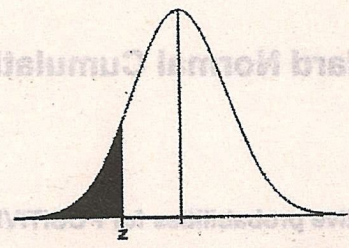
$$\chi^2 = \sum \frac{(o_j - e_j)^2}{e_j}$$

where O_j represents the observed cell frequencies and e_j represents the expected cell frequencies given by

$$e_j = \frac{RC}{N}$$

where R =row total for cell j ; C =column total for cell j ; and N =grand total.

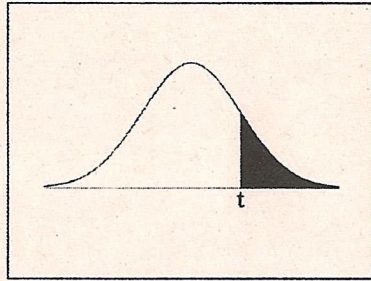
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

t-Distribution Table



The shaded area is equal to α for $t = t_{\alpha}$.

<i>df</i>	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
∞	1.282	1.645	1.960	2.326	2.576

Table A.5 (continued) Critical Values of the Chi-Squared Distribution

v	α									
	0.30	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.466
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.321
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.471	27.688	29.819	34.527
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.124
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.698
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.791
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.819
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.314
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.796
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.619
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.051
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.475
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.994	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.335	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.702
40	44.165	45.616	47.269	51.805	55.758	59.342	60.436	63.691	66.766	73.403
50	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.660
60	65.226	66.981	68.972	74.397	79.082	83.298	84.58	88.379	91.952	99.608

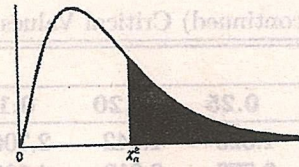


Table A.5 Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.0 ⁴ 393	0.0 ³ 157	0.0 ³ 628	0.0 ³ 982	0.00393	0.0158	0.0642	0.102	0.148	0.455
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.647	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.041	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.878	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336
40	20.707	22.164	23.838	24.433	26.509	29.051	32.345	33.66	34.872	39.335
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335
60	35.534	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335