University of Ruhuna - Faculty of Technology Bachelor of Engineering Technology Honors Degree Level 1(Semester I) Examination, June 2023. Academic Year 2021/2022

Course Unit: TMS1113 Foundation of Mathematics (Written) Duration: 3 hours.

- All symbols have their usual meanings.
- This paper contains five (5) questions on 4 pages.
- Answer all five (5) questions.
- Calculators are not allowed for this examination.

Q1.

a) If
$$a^2 - b^2 = 8$$
 and $ab = 2$; determine the value of $a^4 + b^4$. (10 marks)

b) Solve for the value of x.

i.
$$\log_{\mathbf{x}}(8e^3) = 3 \tag{20 marks}$$

ii.
$$\log_9 x^3 = \log_2 8 \tag{20 marks}$$

c) Determine the value of x of the following equations.

i.
$$x(x-1) = 30$$
 (10 marks)

ii.
$$3x^2 - 5x + 1 = 0$$
 (10 marks)

d) Determine the values of x and y of the following simultaneous equations.

$$x - y = 3$$

 $x^2 - xy - 2y^2 - 7 = 0$ (20 marks)

e) Calculate the exact value of
$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}} - \sqrt{10^{2007}}}$$
. (10 marks)

- a) The angles x and y are acute angles such that $\tan x = \frac{1}{2}$. Further it is given that $\tan(x+y) = 2$. Determine the value of $\sin y$. (20 marks)
- b) Determine the value of x of the equation $\pi 3\cos^{-1}(x+1) = 0$.

(20 marks)

c) Prove the validity of each of the following identities.

(40 marks)

i.
$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta$$

ii.
$$\left(\frac{1+\sin\alpha}{\cos\alpha}\right)^2 + \left(\frac{1-\sin\alpha}{\cos\alpha}\right)^2 = 4(\tan\alpha)^2 + 2$$

d)

i. Prove that
$$1 + \tan^2 x = \sec^2 x$$
. (5 marks)

ii. Hence or otherwise solve the following equation and give a reasonable value for
 x. (15 marks)

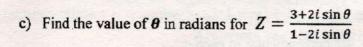
$$2\tan^2 x + \sec^2 x = 5\sec x.$$

,Q3.

a) The complex number **Z** is given as
$$Z = \frac{a+bi}{a-bi}$$
 where $a \in \mathbb{R}$ and $b \in \mathbb{R}$. Show

that
$$\frac{Z^2+1}{2Z} = \frac{a^2-b^2}{a^2+b^2}$$
. (20 marks)

b) It is given that
$$Z = \cos \theta + i \sin \theta$$
. Show that $\frac{2}{1+Z} = 1 - i \tan(\theta/2)$. (40 marks)



(40 marks)

- i. when Z is real
- ii. when Z is imaginary

Q4.

a) Let P be a 2 × 2 matrix such that
$$P = \begin{pmatrix} 4 & -6 \\ -2 & 8 \end{pmatrix}$$
. Find Q if $PQ = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

(20 marks)

b) If A is a 3 × 3 singular matrix, where a is a scalar constant, determine the possible values of a. (30 marks)

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & a \\ 3 & a & 4 \end{pmatrix}$$

c) The 3×3 matrix is given in terms of a constant k.

$$A = \begin{pmatrix} k & 3 & 6 \\ 1 & k & 1 \\ 0 & 4 & 1 \end{pmatrix}$$

i. Show that A has an inverse for all values of k.

(20 marks)

ii. Find A^{-1} in terms of k.

(30 marks)

a) If $\vec{a} = 6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and $\vec{b} = (4p+1)\hat{\imath} + (p-2)\hat{\jmath} + \hat{k}$ where p is a scalar constant, find the value of p if \vec{a} and \vec{b} are perpendicular to each other.

(10 marks)

- b) It is given that $|\vec{u}| = 3$; $|\vec{v}| = 12$ and $|\vec{u}.\vec{v}| = 18$. Show that $|\vec{u} \vec{v}| = 3\sqrt{13}$. (30 marks)
- c) The points A and B have respective positions as follows relative to the origin,

$$A = (2,10,2)$$

$$B = (2,1,2)$$

The angle $A\hat{O}B$ is given as θ .

- i. Show that $\sin \theta = \frac{\sqrt{6}}{3}$. (30 marks)
- ii. Calculate the area of the of the triangle AOB. (10 marks)
- d) A force F is given by the vector $\vec{F} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ is applied on an object which moves from point P = (2,1,0) to point Q = (4,6,2). Find the work done(W = F.D), by the force, where \vec{D} is the displacement. (20 marks)

-----End of the paper-----