Library Copy

University of Ruhuna - Faculty of Technology
Bachelor of Engineering Technology Honors Degree
Level 1(Semester I) Examination, June 2023.
Academic Year 2021/2022

Course Unit: TMS1113 Foundation of Mathematics (Written) **Duration: 3 hours.**

- All symbols have their usual meanings.
- This paper contains five (5) questions on 4 pages.
- Answer all **five (5)** questions.
- Calculators are **not allowed** for this examination.

Q1.

a) If $a^2 - b^2 = 8$ and $ab = 2$; determine the value of $a^4 + b^4$. (10 marks)

b) Solve for the value of x.

i. $\log_x(8e^3) = 3$ (20 marks)

ii. $\log_9 x^3 = \log_2 8$ (20 marks)

c) Determine the value of x of the following equations.

i. $x(x-1) = 30$ (10 marks)

ii. $3x^2 - 5x + 1 = 0$ (10 marks)

d) Determine the values of x and y of the following simultaneous equations.

$$x - y = 3$$

$$x^2 - xy - 2y^2 - 7 = 0$$ (20 marks)

e) Calculate the exact value of $\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}} - \sqrt{10^{2007}}}$. (10 marks)

Q2.

a) The angles $\boldsymbol{x}$ and $\boldsymbol{y}$ are acute angles such that $\tan x = {}^{1}/_{2}$. Further it is given that $\tan(x + y) = 2$. Determine the value of $\sin y$. (20 marks)

b) Determine the value of $\boldsymbol{x}$ of the equation $\pi - 3\cos^{-1}(x + 1) = 0$.

(20 marks)

c) Prove the validity of each of the following identities. (40 marks)

i. $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2cosec\theta$

ii. $\left(\frac{1+\sin\alpha}{\cos\alpha}\right)^2 + \left(\frac{1-\sin\alpha}{\cos\alpha}\right)^2 = 4(\tan\alpha)^2 + 2$

d)

i. Prove that $1 + \tan^2 x = \sec^2 x$. (5 marks)

ii. Hence or otherwise solve the following equation and give a reasonable value for $\boldsymbol{x}$. (15 marks)

$$2\tan^2 x + \sec^2 x = 5\sec x.$$

Q3.

a) The complex number $\mathbf{Z}$ is given as $Z = \frac{a+bi}{a-bi}$ where $a \in \mathbb{R}$ and $b \in \mathbb{R}$. Show that $\frac{Z^2+1}{2Z} = \frac{a^2-b^2}{a^2+b^2}$. (20 marks)

b) It is given that $Z = \cos\theta + i\sin\theta$. Show that $\frac{2}{1+Z} = 1 - i\tan({}^{\theta}/_{2})$.

(40 marks)

c) Find the value of $\boldsymbol{\theta}$ in radians for $Z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$ (40 marks)

i. when Z is real

ii. when Z is imaginary

Q4.

a) Let P be a 2×2 matrix such that $P = \begin{pmatrix} 4 & -6 \\ -2 & 8 \end{pmatrix}$. Find Q if $PQ = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

(20 marks)

b) If A is a 3×3 singular matrix, where a is a scalar constant, determine the possible values of a. (30 marks)

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & a \\ 3 & a & 4 \end{pmatrix}$$

c) The 3×3 matrix is given in terms of a constant k.

$$A = \begin{pmatrix} k & 3 & 6 \\ 1 & k & 1 \\ 0 & 4 & 1 \end{pmatrix}$$

i. Show that A has an inverse for all values of k. (20 marks)

ii. Find A^{-1} in terms of k. (30 marks)

Q5.

a) If $\vec{a} = 6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and $\vec{b} = (4p+1)\hat{\imath} + (p-2)\hat{\jmath} + \hat{k}$ where p is a scalar constant, find the value of p if $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

(10 marks)

b) It is given that $|\vec{u}| = 3$; $|\vec{v}| = 12$ and $|\vec{u}.\vec{v}| = 18$. Show that $|\vec{u} - \vec{v}| = 3\sqrt{13}$.

(30 marks)

c) The points A and B have respective positions as follows relative to the origin,

$$A = (2,10,2)$$

$$B = (2,1,2)$$

The angle $A\hat{O}B$ is given as θ.

i. Show that $\sin\theta = \frac{\sqrt{6}}{3}$. (30 marks)

ii. Calculate the area of the of the triangle AOB. (10 marks)

d) A force F is given by the vector $\vec{F} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ is applied on an object which moves from point $P = (2,1,0)$ to point $Q = (4,6,2)$. Find the work done($W = \boldsymbol{F}.\boldsymbol{D}$), by the force, where $\vec{D}$ is the displacement. (20 marks)

--------**End of the paper**--------