

University of Ruhuna - Faculty of Technology
Bachelor of Engineering Technology Honors Degree
Level 1(Semester I) Examination, June/July 2023.
Academic Year 2021/2022

Course Unit: TMS1152 Applied Calculus I

Duration: 2 hours.

- This paper contains 4 questions on 3 pages.
- Answer **all** questions.
- All symbols have their usual meanings.
- Calculators are **not allowed**.

Q1.

- a) Given that the following simultaneous equations have exactly one pair of solutions, show that $k = \pm 2\sqrt{2}$. (20 marks)

$$\begin{aligned}y - x &= k \\ x^2 + y^2 &= 4\end{aligned}$$

- b) Determine the range of p ; for which the equation $x^2 - 2px + p + 6 = 0$,
- i. Has no real roots
 - ii. Has real roots. (30 marks)

- c) Consider the function $f(x) = \frac{x-1}{\sqrt{x}-1}; x \neq 1$

- i. Determine the limit of the function $f(x)$ as $x \rightarrow 1$. (10 marks)
- ii. What is the domain of the given function? (10 marks)
- iii. What is the range of the of the given function? (10 marks)
- iv. Find $f^{-1}(x)$. (10 marks)
- v. Find $f^{-1}(4)$. (10 marks)

Q2.

Determine the limits of the following functions.

i. $\lim_{t \rightarrow \infty} \sqrt{t^2 + 3} - t$ (20 marks)

ii. $\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$ (20 marks)

iii. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}$ (20 marks)

iv. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$ (20 marks)

v. $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{(x-1)}$ (20 marks)

Q3.

a) Given $y = 4(\sqrt[3]{x^2})$ and $x = w^{-3}$. Determine $\frac{dy}{dw}$. (10 marks)

b) Determine $\frac{dy}{dx}$;

i. $y = e^{x \sin x}$ (10 marks)

ii. $y = \sqrt{\sec(x^2) - 3x}$ (10 marks)

iii. $y = \sin^3(\pi \cos(2x^3))$ (10 marks)

iv. $y = \frac{x^2 \sqrt{7x+3}}{(1+x^2)^5}$; use properties of logarithms to differentiate. (30 marks)

c) Given $y = \frac{\sin x}{1 + \cos x}$, show that the first derivative is equal to $\frac{1}{2} \sec^2\left(\frac{x}{2}\right)$. (30 marks)

Q4.

- a) Calculate the values of a and b if the function $f(x) = ax^2 + bx + 5$ has a tangent at $x = 1$ which is defined by the equation $y = -7x + 3$. (40 marks)
- b) A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radius of the spheres and cylinders are the same. The height of each cylinder is h . The wooden spheres and cylinders are to be painted. (Ignore the holes in spheres and cylinders).
- If the volume of a cylinder is 6 cm^3 , write h in terms of r . (10 marks)
 - Derive an expression for the total surface area S of all the painted surfaces of the necklace. (20 marks)
 - Determine the value of r so that the least amount of paint will be used. (30 marks)

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