UNIVERSITY OF RUHUNA

FACULTY OF GRADUATE STUDIES

Degree of Master of Business Management Semester I End Examination

June 2023

MBM 11033 - Business Mathematics and Statistics

The question paper consists of seven (07) questions.

Answer five (05) questions only.

Non-programmable calculators are permitted. **Duration: Three hours**

01.

i. Simplify the following expression.

$$3c^2 - 2c(c - d) - 3d(c - 2d)$$

(02 marks)

ii. Find the roots of the following quadratic equation.

$$4x^2 - 10x + 5 = 0$$

(04 marks)

iii. Given the following profit function (π) in millions of rupees and output (Q) in units of thousands:

$$\pi = -\frac{Q^3}{3} + 3Q^2 - 5Q - 4$$

a. Calculate the level of output, Q*, that will maximize the profit.

(04 marks)

b. Calculate the maximum profit.

(02 marks)

(Total 12 marks)

02.

i. At the beginning of each quarter, Rs.7500 is deposited into a savings account that pays 8% compounded quarterly. Find the balance in the account at the end of four years.

(04 marks)

ii. Suppose you borrow Rs. 100,000 at 25% compound annual interest rate to be repaid over the next four years. Equal installment payments are required at the end of each year.

a. Determine the annual payment.

(04 marks)

b. Prepare the amortization schedule for the above loan.

(04 marks)

(Total 12 marks)

03.

If $A = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 6 & 8 \\ 9 & 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 2 & 1 \\ 3 & 7 & 5 \\ 2 & 1 & 0 \end{bmatrix}$

You are required to find:

i. A + B

(01 mark)

ii. A – B

(01 mark)

iii. AB

(02 marks)

iv. Determinant of matrix A

(02 marks)

v. Cofactor matrix of A

(02 marks)

vi. Adjoint matrix of A

(02 marks)

vii. Inverse matrix of A

(02 marks)

(Total 12 marks)

04.

i. What is the difference between population and sample?

(02 marks)

ii. The following table shows the sales of packets of milk powder in a retail grocery store over a period of 100 days.

45	85	57	56	74	102	74	54	48	53
80	86	91	102	78	69	96	116	83	122
63	100	89	110	71	92	92	79	117	103
90	101	81	87	113	92	66	106	79	54
58	73	94	86	83	55	90	62	73	66
92	97	80	46	89	69	75	72	104	48
67	89	81	47	105	78	109	67	59	49
67	62	73	106	65	53	98	65	79	95
91	107	58	88	70	123	119	45	73	61
88	61	111	93	89	81	76	91	85	82

Construct a frequency distribution table from the above data, beginning with 45-54, 55-64, ... class intervals with equal width.

(05 marks)

iii. The following table shows the distribution of employees by monthly salary in two similar organizations.

Monthly wages (Rs.)	**Number of Employees**	
	Company A	**Company B**
2000-4000	5	10
4000-6000	12	12
6000-8000	22	15
8000-10000	18	7
10000-12000	4	3

You are required to:

a. Calculate Mean and standard deviation of wages for the two companies.

(04 marks)

b. Which company has the greater variability in wages compared to the average wages?

(01 mark)

(Total 12 marks)

05.

i. What is the relationship between regression analysis and time series analysis?

(02 marks)

ii. The monthly revenue and the electricity expenditure of eight companies are shown in the table below.

Monthly Revenue (Rs. '000)	**Electricity Expenditure (Rs.'000)**
860	58
340	19
770	43
1150	67
480	24
1060	77
620	29
960	48

You are required to:

a. Calculate the least square regression line between monthly revenue and electricity expenses.

(04 marks)

b. Estimate the electricity expenses in rupees when monthly revenue is Rs. 600,000.

(01 mark)

iii. The human resources department of a company is investigating a new procedure for the selection of new management trainees. Applicants are given, prior to appointment, a written test, and a formal interview. The performances of the 10 successful applicants are shown as follows:

Applicant	1	2	3	4	5	6	7	8	9	10
Written Test Marks	90	46	80	40	60	95	85	54	82	70
Interview marks	42	45	75	45	43	70	72	50	55	40

You are required to:

a. Calculate the rank correlation coefficient between the written test and the interview.

(04 marks)

b. Interpret your findings based on the above calculation.

(01 marks)

(Total 12 marks)

06.

i. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of 10 workers chosen at random, four or more will suffer from the disease?

(04 marks)

ii. It is known from past experience that in a certain plant, there are on average 0.25 industrial accidents per month.

a. Find the probability that in a given month, there will be less than three accidents.

b. Find the probability that in a given year there will be more than three accidents.

(04 marks)

iii. A soft drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

a. What fraction of the cups will contain more than 224 milliliters?

b. How many cups will probably overflow if 230 milliliter cups are used for the next 1,000 drinks?

(04 marks)

(Total 12 marks)

07.

i. Distinguish between stratified random sampling and simple random sampling.

(04 marks)

ii. A survey conducted using a sample of 150 banks shows that the average amount spent on setting up a website is Rs.25, 000. The sample standard deviation was Rs.8,000. Construct a 95% confidence interval for the average amount spent on setting up the web by a bank in the population.

(04 marks)

iii. For a large group of students studying accounting, a random sample of 12 students has a mean mark of 57 with a standard deviation of 4. The marks of students are assumed to be normally distributed. Test the hypothesis that the mean mark of all students studying accounting is less than 60. (use a 1% level of significance)

(04 marks)

(Total 12 marks)

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List of formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$I = PRT$$

$$FV_n = P_0(1+i)^n$$

$$n = \frac{ln\left(\frac{FV}{P_0}\right)}{ln\,(1+i)}$$

$$i = \left(\frac{FV}{P_0}\right)^{\frac{1}{n}} - 1$$

$$FVA_n = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$PVA_n = \frac{R}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$PVAD_n = (1+i)\frac{R}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$FVAD_n = R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

$$\bar{X} = A + \left[\frac{\sum fu}{\sum f}\right]^c$$

$$\bar{X} = A + \left[\frac{\sum fd}{\sum f}\right]$$

$$\bar{X} = \frac{\sum fx}{\sum f}$$

$$Median = L_1 + \frac{\left[\frac{n+1}{2} - (\sum f)_1\right]^c}{fm}$$

$$mode = L_1 + \left[\frac{d_1}{d_1 + d_2}\right]^c$$

$$\sigma^2 = C^2\left[\frac{\sum fu^2}{\sum f} - \left[\frac{\sum fu}{\sum f}\right]^2\right]$$

$$\sigma^2 = \frac{\sum fd^2}{\sum f} - \left[\frac{\sum fd}{\sum f}\right]^2$$

$$\sigma^2 = \left[\frac{\sum fx^2}{\sum f} - \bar{X}^2\right]$$

$$\sigma = C\sqrt{\left[\frac{\sum fu^2}{\sum f} - \left[\frac{\sum fu}{\sum f}\right]^2\right]}$$

$$\sigma = \sqrt{\left[\frac{\sum fx^2}{\sum f} - \bar{X}^2\right]}$$

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left[\frac{\sum fd}{\sum f}\right]^2}$$

$$C.V. = \frac{\sigma}{\bar{X}} \times 100$$

$$Y' = a + bx$$

$$b = \frac{n[\sum xy] - [(\sum x)(\sum y)]}{n[\sum x^2] - (\sum x)^2}$$

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$S_{y.x} = \frac{\sum y^2 - a(\sum y) - b(\sum xy)}{n-2}$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r' = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$P\,(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad for\ x = 0,1,2, \ldots and\ \lambda > 0$$

$$S^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1}$$

$$P\,(x) = {}^{n}C_x P^x q^{(n-x)}$$

$$E(X) = \sum x_i \,.\, P(x_i)$$

$$\sigma^2 = E[X^2] - (E[X])^2$$

$100(1-\alpha)\%\ Confidence\ interval$ for parameter(θ)
= Estimator $\pm$ (Reliability Coefficient) $\times$ (Standrad Error)

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \qquad t = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

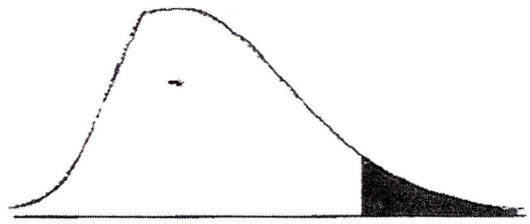
$$z = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

STUDENT'S *t*-DISTRIBUTION

	Level of significance for one-tailed test					
df	*0.100*	*0.050*	*0.025*	*0.010*	*0.005*	*0.0005*
	Level of significance for two-tailed test					
	0.20	*0.10*	*0.05*	*0.002*	*0.01*	*0.001*
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

CRITICAL VALUES OF CHI-SQUARE

This table contains the values of χ^2 that correspond to a specific right-tail area and specific numbers of degrees of freedom df.

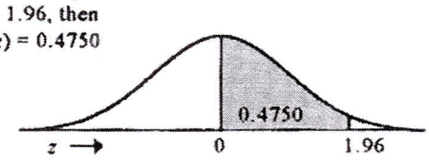


Possible values of χ^2

Degrees of Freedom	Righ-tail Area			
df	*0.10*	*0.05*	*0.02*	*0.01*
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086
6	10.645	12.592	15.033	16.812
7	12.017	14.067	16.622	18.475
8	13.362	15.507	18.168	20.090
9	14.684	16.919	19.679	21.666
10	15.987	18.307	21.161	23.209
11	17.275	19.675	22.618	24.725
12	18.549	21.026	24.054	26.217
13	19.812	22.362	25.472	27.688
14	21.064	23.685	26.873	29.141
15	22.307	24.996	28.259	30.578
16	23.542	26.296	29.633	32.000
17	24.769	27.587	30.995	33.409
18	25.989	28.869	32.346	34.805
19	27.204	30.144	33.687	36.191
20	28.412	31.410	35.020	37.566
21	29.615	32.671	36.343	38.932
22	30.813	33.924	37.659	40.289
23	32.007	35.172	38.968	41.638
24	33.196	36.415	40.270	42.980
25	34.382	37.652	41.566	44.314
26	35.563	38.885	42.856	45.642
27	36.741	40.113	44.140	46.963
28	37.916	41.337	45.419	48.278
29	39.087	42.557	46.693	49.588
30	40.256	43.773	47.962	50.892

AREAS UNDER THE NORMAL CURVE

Example
If $z = 1.96$, then
$P(0 \text{ to } z) = 0.4750$



Z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990