OPTIMIZING THE WAREHOUSE LOCATION AND DISTRIBUTOR ALLOCATION: A CASE STUDY OF THE LPG DISTRIBUTION IN SRI LANKA

Chamari I. Kithulgoda[a] and D.D.M.Jayasundara[b]

[a]*Department of Computer Science, University of Ruhuna, Matara, Sri Lanka*
chamari@dcs.ruh.ac.lk

[b]*Department of Statistics & Computer Science, University of Kelaniya, Kelaniya, Sri lanka*
jayasund@kln.ac.lk

Abstract
In this paper, we deal with a real world warehouse location and distributor allocation problem of Liquefied Petroleum Gas (LPG) distribution in Sri Lanka. The existing supply chain has a single storage plant and 33 different distributors throughout the country. With this system, approximately 65 km (6.4946e+004 m) should be travelled to satisfy a unit demand. We choose to proceed with the P-median model, which locates "p" facilities among "n" demand points and allocates each demand point to one of the facilities by assuming every demand point can be elected as a median. Then the problem was solved by computational Myopic algorithm and the computational Lagrangian algorithm. As the first median, both Myopic and Lagrangian algorithms selected the same distributor node "no.16 " as the warehouse with the average distance of 5.2926e+004m to satisfy a unit demand. In the case of selecting the two medians, while the Myopic algorithm proposed the node "no.16" and node "no.18" as best locations with 3.7918e+004m average travelling distance, the Lagrangian algorithm selected node "no.18" and node "no.06" as best locations with 3.6673e+004m average distance. In later case, optimum demand point allocation could be done by assigning nodes 1,2,3,4,7,14,18,19,20,21,23 and 28 to the warehouse which will be located at node "no.18" and nodes 5,6,8,9,10,11,12,13,15,16,17,22,24,25,26,27, 29,30,31,32 and 33 to the warehouse which will be located at node "no.16" .The resulted computerized user interface provides drop down menu to select the number of warehouses to be located and then outputs the best nodes to be elected as warehouses and displays the best possible demand point allocation method.

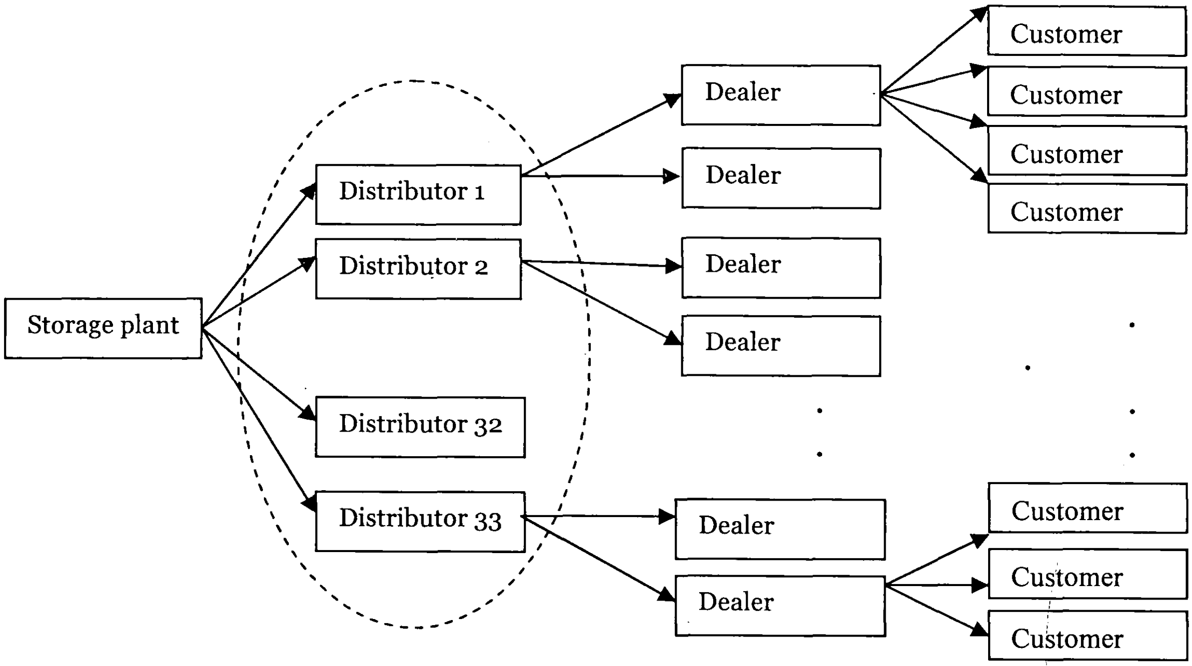
Keywords: Facility Location; Optimization; Lagrangianalgorithm; P-Median Model; Supply Chain

1. Introduction

Optimum resource utilization can be considered as animportant phase of the supply chain management."Where to locate plants and distribution centres" is a location decision which is critical and most difficult of the decisions needed to realize an efficient supply chain. These location decisions are often fixed and difficult to change even in the intermediate term. Inefficient locations for production and assembly plants as well as distribution centres will result in excess costs being incurred throughout the lifetime of the facilities (Daskin, Snyder &Berger, 2003).

The existing LPG supply chain has single storage plant and 33 different distributors out of its existing dealers who help to meet the needs of over 1500 dealers throughout the country as shown in figure 1. As a consequence of this centralized distribution system number of inconveniences such as, significant waiting time at the vehicle queue, high transportation cost and inefficiencies due to weak highway system were emerged. The importance of reconfiguring the distribution network based on decentralized distribution strategy is highlighted.

Figure 1:LPG Supply Chain in Sri Lanka (area of our interest is displayed by the dashed oval)



According to Chopra and Meindl (2003, p.99) supply chain network design decisions should answer questions; what role should each facility play? , where should facilities be located? , how much capacity should be allocated to each facility? , what markets should each facility serve? and which supply sources should feed each facility?.In our research, we focused on obtaining optimum solutions for the aforementioned second and fourth questions regarding to facility location and capacity allocation decisions by programmed Myopic algorithm and highly mathematical Lagrangian algorithm.

We can summarize the objectives of this study as, proposing sites to locate warehouses, and allocating them into demand points, so as to minimize the total demand weighted distance between customers (demand points) and facilities (warehouses). Explicitly, it is expected to select distributors who are to be converted into warehouses and allocate those warehouses to remaining distributors in optimum manner.

2. Materials and Methods

Our work uses P-median model, which locates "p" facilities among "n" demand points and allocates each demand point to one of the facilities by assuming every demand point can be elected as a median. More specifically, we locate "p" warehouses among 33 distributors for serving "(33-p)" distributors while minimizing the sum of the shortest demand weighted distance between warehouses and distributors.

2.1 P-median Problem

Mathematically, the P-median problem can be summarized as follows (Daskin 1995,p.200-201):

Inputs

h_i =demand at node i

d_{ij} =distance from node i to candidate site j

P=number of facilities to locate

Decision variables

$$X_j = \begin{cases} 1 & \text{if we locate at candidate node } j \\ 0 & \text{if not} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demand at node } i \text{ are served by facility at node } j \\ 0 & \text{if not} \end{cases}$$

With this notation P-median problem can be formulated as:

Minimize $\sum_i \sum_j h_i d_{ij} Y_{ij}$ $\sum_i \sum_j \boldsymbol{h_i}\, \boldsymbol{d_{ij}} \boldsymbol{Y_{ij}}$ (a)

Subject to $\sum_j Y_{ij} = 1 \quad \forall i$ (b)

$\sum_j X_j = P$ (c)

$Y_{ij} - X_j \le 0 \quad \forall i,j$ (d)

$X_j = 0,1 \quad \forall j$ (e)

$Y_{ij} = 0,1 \quad \forall i,j$ (f)

The objective function (a) minimizes the total demand weighted distance between each demand node and the facility nearest to it.Constraint (b) specifies that each demand node i is assigning to one and only facility j.Constraint (c) states that exactly P facilities are to be located.Constraint (d) links the two decision variables. In essence, it requires that demand at node$i$ can only be assign to facility j, if there is a facility located at j. Constraint (e) and (f) ensure standard integrality conditions .

2.2 Myopic Algorithm for the P-median problem

The Myopic algorithm comes under the class of construction heuristics. Daskin (1995,p.209) statedthe followingapproach.

Step1:Initialize k=0 (k will count the number of facilities we have located so far) and $X_k = \phi$, the empty set (X_k will give the location of the k facilities that we have located at each stage of the algorithm).

Step2: Increment k, the counter on the number of facilities located.

Step3: Compute $Z_j^k = \sum_i h_i d(i, j \cup X_{k-1})$ for each node j which is not in the set X_{k-1} .Note that Z_j^k gives the value of the P-median objective function if we locate the $k$thfacility at node j, given that the first k-1 facilities are at the locations given in the set X_{k-1} (and node j is not part of that set).

Step4:Find the node $j^*(k)$ that minimizes Z_j^k, that is, $j^*(k)$ = argmin $_j\{Z_j^k\}$.Note that $j^*(k)$ gives the best location for the kth facility, given the location of the first k-1 facilities. Add node $j^*(k)$ to the set X_{k-1} to obtain the set X_k ; that is, set $X_k = X_{k-1} \cup j^*(k)$.

Step5: If $k=P$ (i.e., we have located P facilities), stop; the set X_p is the solution to the myopic algorithm. If $k<P$, go to step 2.

2.3 Lagrangian Relaxation Algorithm

Following steps of Lagrangian relaxation algorithm can be adapted from Daskin (1995,p. 221-225).

Step 1: Relax one or more constraints by multiplying those by Lagrange multipliers and bringing the constraint(s) into objective function.By relaxing the constraint (b) and introducing variables called Lagrange multipliers (λ_i), the objective function is modified and obtain the relaxed objective function.

$$\underset{\lambda}{MAX}\ \underset{X,Y}{MIN} \sum_i \sum_j h_i d_{ij} Y_{ij} + \sum_i \lambda_i [1 - \sum_j Y_{ij}] = \sum_i \sum_j (h_i d_{ij} - \lambda_i) Y_{ij} + \sum_i \lambda_i \quad \text{(a')}$$

Step2:solve the relaxed problem by minimizing the objective function (a') for fixed values of the Lagrange multipliers (λ_i) subject to the constraints (b),(c),(d),(e) and (f).

Step3: Begin by computing the value of setting each of the X_j values to 1. This value is given by $V_j = \sum_i \min\{0, h_i d_{ij} - \lambda_i\}$ for each candidate location j.Find the P smallest values of V_j and set the corresponding location variables $X_j = 1$ and all other location variables $X_j = 0$. Then set:

$$Y_{ij} = \begin{cases} 1 & \text{if } X_j = 1 \text{ and } h_i d_{ij} - \lambda_i < 0 \\ 0 & \text{if not} \end{cases}$$

Step 3: By ignoring the allocation variables, Y_{ij}, and setting the facilities at those sites for which $X_j = 1$, find a primal feasible solution related to the Lagrangian solution. We then can let S= {j| $X_j = 1$}; that is, S is the set of facility locations. For each demand node i, we then find $\hat{j}_i = \arg\min_{j\in s}\{d_{ij}\}$, that is, $\hat{j}_i$ is the open facility that is closest to node i. We then set $\hat{Y}_{ik} = 1$ if k = $\hat{j}_i$ and $\hat{Y}_{ik} = 0$ for all other locations k as before.(Note that we are using $\hat{Y}_{ik}$ to denote the feasible allocation of demands that results from the use of the location decisions in the relaxed problem.)Then evaluate the P-median objective function, $\sum_i \sum_j h_i d_{ij} \hat{Y}_{ij}$. This value is an upper bound on the solution. Clearly, the best such (smallest) value over all iterations of the Lagrangian relaxation procedure should be used as the upper bound.

Step 4: Compute a step size, t^n, at the nth iteration of the Lagrangianprocedure and then modify the Lagrange multiplier.

$$t^n = \alpha^n (UB - L^n) / \sum_i \{\sum_j Y_{ij}^n - 1\}^2 \quad \text{(g)}$$

Where,

t^n= the step size at the n^{th} iteration of the Lagrangian procedure

α^n = a constant on the n^{th} iteration, with α^1 generally set to 2

UB = the best upper bound on the P-median objective function

L^n = the objective function of the Lagrangian function (a) on the nth iteration

Y_{ij}^n = the optimal value of the allocation variable, Y_{ij}, on the nth iteration

The Lagrange multipliers are then updated according to the following equation:

$$\lambda_i^{n+1} = \max\{0, \lambda_i^n - t^n(\sum_j Y_{ij}^n - 1)\}$$

Restrict the Lagrange multipliers to nonnegative values as long as all demands, h_i, and all distances, d_{ij}, are nonnegative. Obtain values of the lower bounds from the function (a').

Step 5: If at any point in time, the lower bound is equal or very close to the upper bound, the optimal solution to the original problem is obtained and the algorithm terminates. The values of αshould not be less than 0.00005. If the lower bound has not decreased after four consecutive iterations,set α toα /2. If none of the above stopping conditions are met, the implementation reiterates starting at Step 2.

2.4 Technical Computing

Even though the backgrounds behind these algorithms are highly mathematical, we propose graphical user interface to bridge complex computer programming codes to comprehensible interactive window. The interfaces were built with the aid of MATLAB graphical user interface development environment.

2.5 Input Data

A basic element in the formulation of location problems is the concept of distance. The distance between two location points (x_i, y_i) and (x_j, y_j)is calculated by:

$d_{ij} = [(x_j - x_i)^2 + (y_j - y_i)^2]^{1/2}$

We represent 33 different distributors' locations by nodes 1, 2, 3...33 and the existing storage plant by node "no.0'. Distances (in meters) from storage plant (node "no.0") to each distributor node *i* (where *i*=1, 2, 3, ..., 33) are :

do1	=	195,310	do10	=	13,850	do19	=	59,600	do28	=	89,290
do2	=	135,610	do11	=	104,600	do20	=	78,310	do29	=	26,360
do3	=	159,770	do12	=	13,280	do21	=	53,490	do30	=	6,940
do4	=	122,190	do13	=	16,170	do22	=	58,570	do31	=	6,910
do5	=	11,870	do14	=	300,240	do23	=	91,850	do32	=	16,520
do6	=	6,310	do15	=	5,570	do24	=	241,550	do33	=	56,880
do7	=	212,160	do16	=	4,390	do25	=	173,820			
do8	=	9,020	do17	=	39,830	do26	=	31,280			
do9	=	71,060	do18	=	83,520	do27	=	98,740			

Annual demand (in kilograms) of each distributor node *i*,

h1	=	104352
h2	=	210456
h3	=	316608
h4	=	186456
h5	=	279456
h6	=	342912
h7	=	154656
h8	=	220608
h9	=	131808

h10	=	333912	h19	=	280608	h28	=	279456
h11	=	323760	h20	=	273408	h29	=	422064
h12	=	453216	h21	=	469824	h30	=	537216
h13	=	260064	h22	=	258912	h31	=	226608
h14	=	103660.8	h23	=	304608	h32	=	380064
h15	=	485520	h24	=	280608	h33	=	236299.2
h16	=	345216	h25	=	320064			
h17	=	255456	h26	=	282912			
h18	=	522960	h27	=	378912			

The average distance can be calculated by the following equation.

Average distance $=Z_j^n$ / Total demand in the network

Here,

n = number of medians

Z_j^n = value of the P-median objective function if we locate the n^{th}median at node j

3. Results

Since the existing centralized distribution network has its one and only storage plant at node "no.0" , the average distance travel = 6.4946e+004 m. Approximately 65 km should be travelledto satisfy a unit demand. In our work, up to 10 number of nodes are selected which are to be converted into warehouses by two algorithms namely Myopic and Lagrangian. The table 1 depicts the results for first 10 Myopic medians for the LPG distribution network.

Table 1.Results Obtained from the Myopic Algorithm

Number of Warehouses to be Proposed	Average Distance to Travel * 10^4 m	Selected Node/s
1	5.2926	16
2	3.7918	16,18
3	3.0714	16,18,24
4	2.5547	16,18,24,3
5	2.2829	16,18,24,3,7
6	2.0232	16,18,24,3,7,22
7	1.7821	16,18,24,3,7,22,27
8	1.5689	16,18,24,3,7,22,27,21
9	1.4098	16,18,24,3,7,22,27,21,14
10	1.2738	16,18,24,3,7,22,27,21,14,28

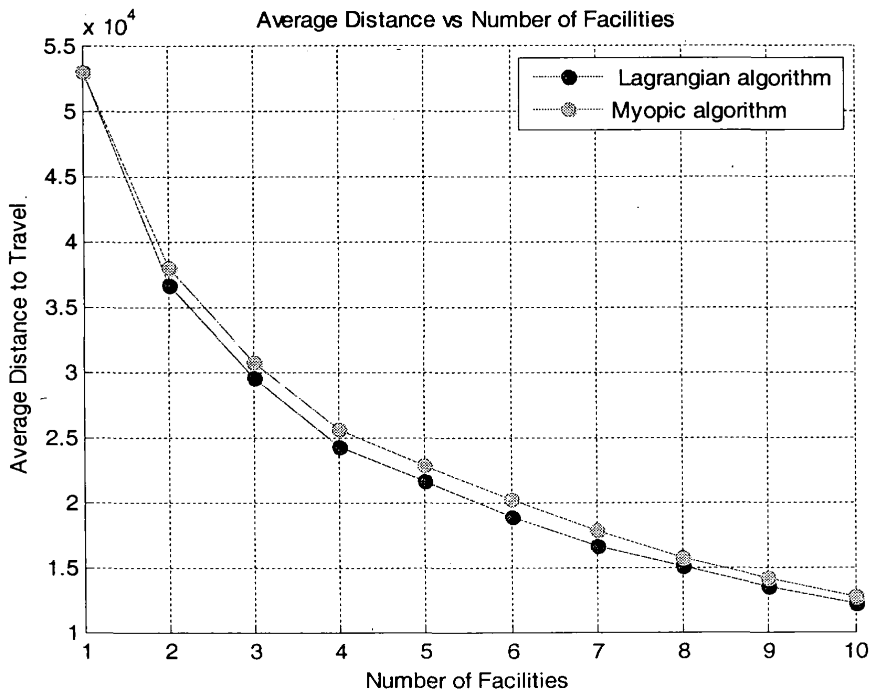
Note that the Myopic algorithm holds the selected nodes which are to be converted to warehouses of the previous stage and pass those to the next step. Selected nodes and average distance for each solution of Lagrangian algorithm is given in table2.

Table 2.Results Obtained from the Lagrangian Algorithm

Number of Warehouses to be Proposed	Average Distance to Travel * 10^4 m	Selected Node/s
1	5.2926	16
2	3.6673	18, 6
3	2.9481	24, 18, 31
4	2.4314	18, 3, 24, 31
5	2.1596	3, 24, 18, 7, 31
6	1.8918	3, 18, 24, 7, 22, 6
7	1.6684	3, 24, 18, 21, 7, 22, 6
8	1.5093	3, 24, 21, 18, 7, 22, 14, 6
9	1.351	3, 21, 24, 22, 7, 14, 18, 27, 15
10	1.215	3, 21, 14, 7, 22, 15, 24, 18, 27, 28

The figure 2 compares solutions obtained from two algorithms. As the first median, when the number of warehouses to be located equals to one, both Myopic and Lagrangian algorithms select the same distributor node "no.16 " as the best location with the average distance of 5.2926e+004m to satisfy a unitdemand. Then,Lagrangian algorithm always results better average distances with respect to the Myopic algorithm.

Figure2:Comparison between Results Obtained from two Algorithms



Since, Lagrangian algorithm suggests better result set, assuming that those locations are converted into warehouses the following optimal warehouse allocation method (table 3) has been suggested.Each row of the table 3 illustrates number of warehouses to be located, selected nodes to be converted into warehouses and the best demand nodes allocation mechanism respectively. For instance, in the case of selecting the two medians, optimum demand point allocation could be done by assigning nodes 1,2,3,4,7,14,18,19,20,21,23 and 28 to the warehouse which will be located at node "no.18" and nodes

5,6,8,9,10,11,12,13,15,16,17,22,24,25,26,27, 29,30,31,32 and 33 to the warehouse which will be located at node "no.16".

Table 3.Optimum Warehouse Allocation

Number of Warehouses	Selected Node/s	Demand Nodes Allocation								
		1	2	3	4	5	6	7	8	9
1	16	16	16	16	16	16	16	16	16	16
2	18, 6	18	18	18	18	6	6	18	6	6
3	24, 18, 31	18	24	18	18	31	31	18	31	31
4	18, 3, 24, 31	18	24	3	18	31	31	18	31	31
5	3, 24, 18, 7, 31	7	24	3	18	31	31	7	31	31
6	3, 18, 24, 7, 22, 6	7	24	3	18	6	6	7	6	22
7	3, 24, 18, 21, 7, 22, 6	7	24	3	18	6	6	7	6	22
8	3, 24, 21, 18, 7, 22, 14, 6	7	24	3	18	6	6	7	6	22
9	3, 21, 24, 22, 7, 14, 18, 27, 15	7	24	3	18	27	27	7	27	22
10	3, 21, 14, 7, 22, 15, 24, 18, 27, 28	7	24	3	28	27	27	7	27	22

Contd..

Number of Warehouses	Selected Node/s	Demand Nodes Allocation								
		10	11	12	13	14	15	16	17	18
1	16	16	16	16	16	16	16	16	16	16
2	18, 6	6	6	6	6	18	6	6	6	18
3	24, 18, 31	31	24	31	31	18	31	31	31	18
4	18, 3, 24, 31	31	24	31	31	3	31	31	31	18
5	3, 24, 18, 7, 31	31	24	31	31	3	31	31	31	18
6	3, 18, 24, 7, 22, 6	6	24	6	6	3	6	6	6	18
7	3, 24, 18, 21, 7, 22, 6	6	24	6	6	3	6	6	6	18
8	3, 24, 21, 18, 7, 22, 14, 6	6	24	6	6	14	6	6	6	18
9	3, 21, 24, 22, 7, 14, 18, 27, 15	27	24	27	15	14	15	15	27	18
10	3, 21, 14, 7, 22, 15, 24, 18, 27, 28	27	24	27	15	14	15	15	27	18

Contd..

Number of Warehouses	Selected Node/s	Demand Nodes Allocation								
		19	20	21	22	23	24	25	26	27
1	16	16	16	16	16	16	16	16	16	16
2	18, 6	18	18	18	6	18	6	6	6	6
3	24, 18, 31	18	18	18	31	18	24	31	31	31
4	18, 3, 24, 31	18	18	18	31	18	24	31	31	31
5	3, 24, 18, 7, 31	18	18	18	31	18	24	31	31	31
6	3, 18, 24, 7, 22, 6	18	18	18	22	18	24	6	22	6
7	3, 24, 18, 21, 7, 22, 6	21	18	21	22	18	24	6	22	6
8	3, 24, 21, 18, 7, 22, 14, 6	21	18	21	22	18	24	6	22	6
9	3, 21, 24, 22, 7, 14, 18, 27, 15	21	18	21	22	18	24	15	15	27

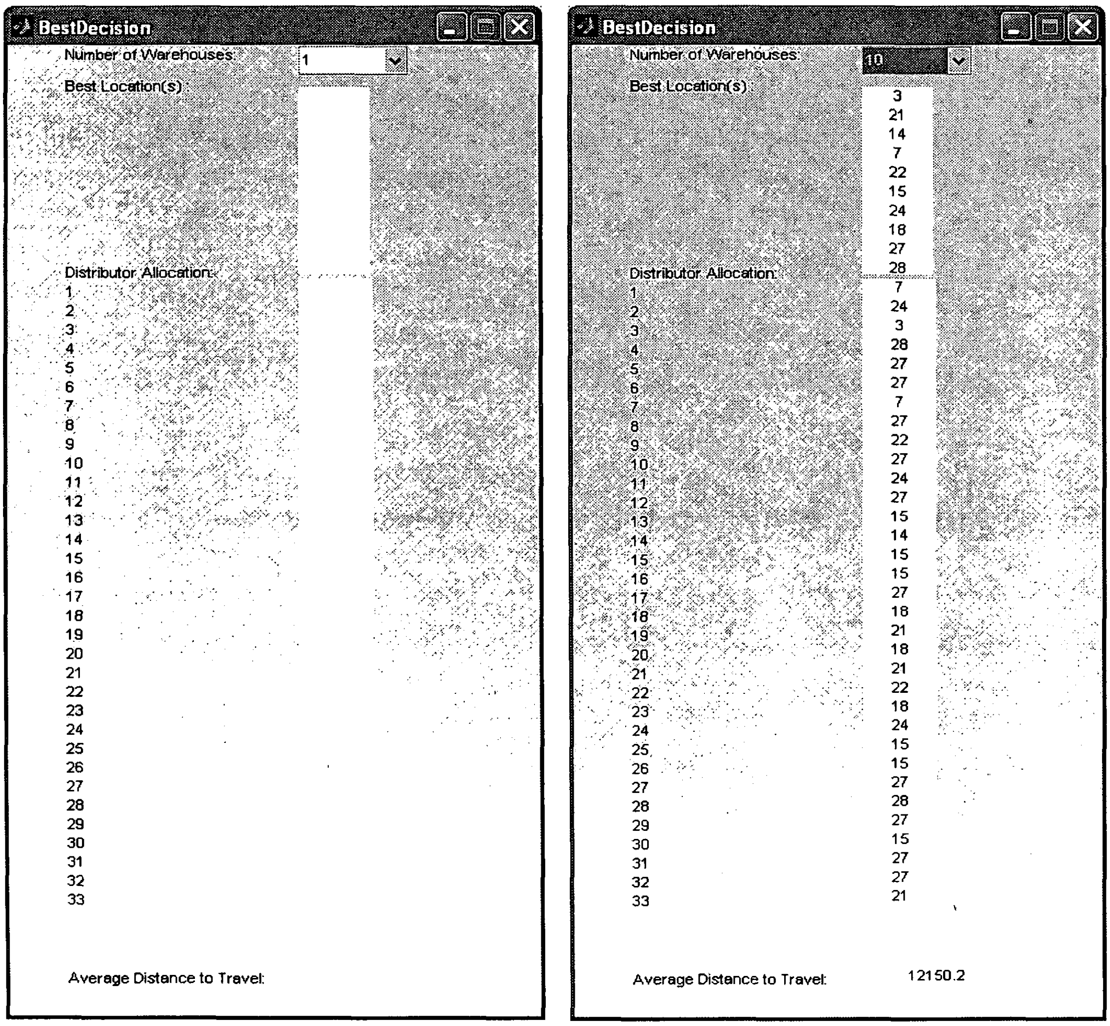
10	3, 21, 14, 7, 22, 15, 24, 18, 27, 28	21	18	21	22	18	24	15	15	27

Contd..

Number of Warehouses	Selected Node/s	Demand Nodes Allocation					
		28	29	30	31	32	33
1	16	16	16	16	16	16	16
2	18, 6	18	6	6	6	6	6
3	24, 18, 31	18	31	31	31	31	31
4	18, 3, 24, 31	18	31	31	31	31	31
5	3, 24, 18, 7, 31	18	31	31	31	31	31
6	3, 18, 24, 7, 22, 6	18	6	6	6	6	6
7	3, 24, 18, 21, 7, 22, 6	18	6	6	6	6	21
8	3, 24, 21, 18, 7, 22, 14, 6	18	6	6	6	6	21
9	3, 21, 24, 22, 7, 14, 18, 27, 15	18	27	15	27	27	21
10	3, 21, 14, 7, 22, 15, 24, 18, 27, 28	28	27	15	27	27	21

The following figure 3 shows the computerized user interface which provides drop down menu to select the number of warehouses to be located and then outputs the best nodes to be elected as warehouses and displays the best possible demand point allocation method.

Figure 3:Graphical User Interfacebefore and after Inserting Number of Warehoused to be Located



4. Conclusions and Future Directions

Our research work successfully addressed two supply chain network design questions, where should warehouses be located? and which warehouse should feed eachdistributor?. More specifically, distributors who are to be converted into warehouses were selected through two different algorithms and those warehouses were allocated to distributors in optimum manner. Comprehensible, interactive GUI facilitates by keeping decision makers away from complex mathematical location models and programming codes.

Even though the solutions given by Myopic algorithm may not be the optimal in some cases, simplicity of this algorithm forces to use this commonly. In addition, if there are facilities those are already established and cannot be relocated, while we are in need of locating one additional facility, Myopic algorithm approach is fruitful.

Obviously, more number of facilities ensure more benefits to their customers and less average distance to travel , but higher the facility costs. Since location decisions are long term and strategic , uncertainty of location model parameters should also be considered.Although the distance between twodistributor locations is not the straight line distance in real world, obtaining distance along highways between given locations is also impractical in nature.

References

Altekar, R. V. (2005).*Supply chain management concepts and cases*, New Delhi,India: PHI Learning Pvt. Ltd.

Arya,V.,Garg,N.,Khandekar,R., Meyerson,A.,Munagala,K. &Pandit,V.,(2001).*Local search heuristics for facility location problems.* Retrieved June 15, 2010, from http://www.cs.ucla.edu/~awm/papers/lsearch.ps

Ballou, R. H. (2003). *Business Logistics: Supply Chain Management* (5th ed.). New Jersey, USA: Prentice Hall.

Chopra, S., &Meindl, P. (2004). *Supply Chain Management* (2nd ed.). Delhi, India: Pearson Education,Inc.

Coyle,J.J.,Bardi,E.J.,&Langley,C.J.(2003).The Management of Business logistics:A Supply Chain Perspective(7th ed.).USA:South-Western.

Creating Graphical User Interfaces in MATLAB *(2012)*Retrieved January 10, 2012, from http://www.mathworks.in/discovery/matlab-gui.html

Daskin, M. S. (1995). *Network and Discrete Location Models, Algorithms, and Applications.* New York, USA: Wiley & Sons.

Daskin,M.S.,Snyder,L.V.,&Berger R.T.(2003). *Facility Location inSupply Chain Design.*RetrievedMay 10, 2010, from http://www.springerlink.com/index/H541677534645P14.pdf

Eroglu, E.,&Keskinturk,T.(2006).*Warehouse location problem with genetic algorithm,*Retrieved June 20, 2010,fromhttp://www.academia.edu/229116/ Warehouse_Location_Problem_with_Genetic_Algorithm

Falcone,M.A.,Lopes,H.S. &Coelho,L.S., *'Supply chain optimization using evolutionary algorithms',Journal of Computer Applications in Technology Vol.31,Nos 3/4,* Retrieved May 28, 2010, from http://www.produtronica.pucpr.br/sip/conteudo/ProdBibliog/Leandro/Supply-artigo31.pdf

Grant, D.B.,Lambert,D.m.,Stock,J.R., &Ellram, L.M.(2006).*Fundamentals of Logistics Management,* Berkshire, England: McGraw-Hill Higher Education.

Jacobs,F.R.,Chase,R.B.,&Aquilano,N.J.(2009).Operations and Supply Management(12th ed.).New York,USA:McGraw-Hill.

Klose, A., &Drexl, A.(2005).*Facility location models for distribution system design.* Retrieved June 26, 2010, from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.83.7234&rep=rep1&type=pdf

Klose,A.&Drexl,A.(2003).*Facility location models for distribution system design.* Retrieved May 15 , 2010, from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.83.7234&rep=rep1&type=pdf

Love,R.F., Morris, J.G.,Wesolowsky,G.O. (1988). *Facilities Location Models & Methods.* New York, USA: North-Holland.

Melo,M.T.,Nickel,S.,&Saldanha-da-Gama,F.(2009).*Facility location and supply chain management-A review* . Retrieved April 25, 2010, from http://132.70.58.206/_Uploads/dbsAttachedFiles/pharmaReview.pdf

Menzer,J.T.(2000).*Supply Chain Management*(2nd ed.).New Delhi,India:SAGE.

Rardin, R. L. (1998). *Optimization in operations research.* New Jersey, USA: Prentice Hall.

Skjott-Larsen,T.(2007).*Managing the Global Supply Chain*(3rd ed.).DK:Copenhagen Business School press.

Suomalainen, E.(2006). *Multicriteria analysis and visualization of location allocation problems*, Retrieved June 25, 2010,from http://www.researchgate.net/publication/ 46257356_Multicriteria_analysis_and_ visualizationof_location-allocation_problems

Zandin, K. B., & Maynard, H. B. (2001). *Maynard's Industrial Engineering Handbook* (5th ed.). New York, USA: McGraw-Hill Professional.