

University of Ruhuna

2018/2019 Academic Year - BA (Special) Degree – 2000 Level

First Semester Examination – 2019 October/November

STS21623 – Intermediate Mathematics (English Medium)

Answer 4 questions.

(Calculators are not allowed)

Duration: 03 Hours

(1). Differentiate following function with respect to x .

(i) $y = 3x^5 + 4x^4 - 5x^3 - 2x^2 + x - 6$ (1 mark)

(ii) $y = \frac{x-1}{x^2+1}$ (1 mark)

(iii) $y = (3x - 4)(5x^2 + 1)$ (1 mark)

(iv) $y = (2x^2 - 3)^2(3x + 5)$ (2 mark)

(v) $y = \log(4x^3 - 2x^2 + 2x)$ (1 mark)

(vi) $y = \log(x^2 - 2x + 1)^2$ (2 marks)

(vii) $y = e^{3x^3-4}$ (1 mark)

(viii) $y = (2x^2 - 3)e^{4x}$ (2 marks)

(ix) $y = \cos^2(x^3 - x)$ (2 marks)

(x) $y = \tan(x^3 + 3)^3$ (2 marks)

(2). (a). Find implicit differentiations of the following functions.

(i) $y^3 - 2xy + 5 = 0$ (2 marks)

(ii) $y^2x - x^2y + y - x + 4 = 0$ (2 marks)

(b). Find second order derivatives of the following functions with respect to x .

(i) $y = 2x^4 + 3x^3 + 5x^2 - 4x$ (2 marks)

(ii) $y = e^{-x^3+2}$ (2 marks)

(c). Find total differentials of the following functions.

(i) $Z = 2x^3 - 7xy + 5y^2$ (2 marks)

(ii) $Z = e^{-x^2y+5x}$ (2 marks)

(d). if $Z = x^2y - xy^2$, $x = at^2$, and $y = 2at$ then find $\frac{dZ}{dt}$. (3 marks)

(3). (a). Solve following integrals.

- (i). $\int 4x^3 dx$ (marks 1)
- (ii). $\int 2x(x^2 - 5)^2 dx$ (marks 1)
- (iii). $\int e^{2x} dx$ (marks 1)
- (iv). $\int (2e^x - \frac{3}{x^4}) dx$ (marks 1)
- (v). $\int x^2 e^{3x} dx$ (marks 3)
- (vi). $\int x(x^2 + 4)^2 dx$ (marks 1)
- (vii). $\int \frac{10x-5}{x^2-x+1} dx$ (marks 1)
- (viii). $\int \sec^2 x dx$ (marks 1)

(b). Evaluate following definite integrals.

- (i). $\int_1^2 (x^3 + 3x)^2 (x^2 + 1) dx$ (marks 3)
- (ii). $\int_0^\pi \sin^2 x dx$ (marks 2)

(4). (a). Sketch $y = \frac{9x}{x^2+9}$ curve using differentiation. (marks 10)

(b). The daily profit, P , of a factory is given by $P = 8x - 0.02x^2$, where x is the number of products of daily manufactured by the factory. How many products will give maximum profit and what is the maximum profit? (marks 5)

(5). (a). Find the area bounded by the curve $y = x^2 + x + 4$, the x -axis and the ordinates $x = 1$ and $x = 3$. (marks 7)

(b). Find the area enclosed by the curve $y = x - x^2$, the x -axis, and the ordinates $x = 0$ and $x = 2$. (marks 8)

(6). (a). find the scalar product and vector product of following \underline{a} and \underline{b} two vectors.

$$\underline{a} = 3\underline{i} + 2\underline{j} - 4\underline{k}, \text{ and } \underline{b} = 5\underline{i} - 3\underline{j} + \underline{k}.$$

What is the angle between \underline{a} and \underline{b} two vectors? (marks 6)

(b). Find a unit vector which is perpendicular to both of the vectors $\underline{a} = \underline{i} + 3\underline{j} - 2\underline{k}$, and $\underline{b} = 4\underline{i} - 2\underline{k}$. (marks 4)

(c). Find the area of the parallelogram with edges represented by the vectors $2\underline{i} - \underline{j} + 3\underline{k}$ and $7\underline{i} + \underline{j} + \underline{k}$. (marks 5)

(7). (a). Evaluate the following limits.

(i). $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$ (marks 3)

(ii). $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$ (marks 1)

(iii). $\lim_{x \rightarrow 3} \frac{\sqrt{12-x} - x}{\sqrt{6+x} - 3}$ (marks 3)

(iv). $\lim_{x \rightarrow \infty} \frac{x-1}{2x^2+3}$ (marks 2)

(b). Solve following system of equations using matrices.

$$2x - 3y + z = -3$$

$$3x + y - 3z = 6$$

$$x + 2y + 2z = 3$$

(marks 6)

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