UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: March 2023

Module Number: ME 3305 **Module Name: Modelling and Controlling of Dynamic Systems**

[**Three** Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:

Some necessary equations and a partial table of Laplace transformation pairs have been provided on the question paper. **You may make additional assumptions**, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

The standard form of a second-order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;

$T_s = \frac{4}{\zeta\omega_n}$ ($\pm 2\%$ settling time);

Percentage Overshoot = $e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

Q1. Figure Q1 shows a mechanical dumbbell. The mass of each steel ball is m and balls are connected by a solid bar having zero mass. The length from the centre of one ball to the other is 2r. An elastic string is attached to the centre of the dumbbell, as shown, acts as a rotational spring (with spring constant k) and the moment of inertia of the dumbbell about the elastic string is $J = 2mr^2$. In the air, the damping coefficient of the dumbbell is b. The dumbbell is twisted (rotated) about elastic string up to θ_0 at $t = 0$.

a) Obtain the differential equation describing untwisting of the dumbbell when it is let go from initial condition θ_0.

[1 Mark]

b) Obtain the Laplace transform of (a) above to describe $\theta(s)$ in terms of initial condition θ_0, natural frequency ω_n,and damping ratio ζ, where $2\zeta\omega_n = {}^{b}/_{J}$ and $\omega_n^2 = {}^{k}/_{J}$.

[2 Marks]

c) Obtain the $\theta(t)$ by taking the inverse Laplace transform of (b) above and show that the untwisting motion is sinusoidal.

[3 Marks]

d) Extract and state the equation of the exponential envelope of decay from (c) above.

[1 Mark]

e) Obtain the values of ω_n and ζ assuming $r = 0.5\ m,\ m = 1\ kg,\ b = 0.0002\ Nms/rad,\ k = 0.0002\ Nm/rad.$

[2 Marks]

f) If the initial twist, $\theta_0 = 4000\ degrees$, obtain the time required to decay the amplitude of sinusoidal oscillation to 10 degrees.

[3 Marks]

Q2. a) Figure Q2 shows an electrical system where output of the system is $v_o(t)$ when input is $v_i(t)$.

i. Obtain the transfer function of the system in Laplace domain, describing $\frac{v_2}{v_i}$ in terms of C and R_3

[2 Marks]

ii. Obtain the transfer function of the system describing $\frac{v_o}{v_i}$ in Laplace domain assuming gain of the op-amp is A.

[2 Marks]

iii. If the operational amplifier in figure Q2 is an ideal op-amp, obtain the transfer function of the system describing $\frac{v_o}{v_i}$ in Laplace domain, using the answer of (ii) above.

[2 Marks]

iv. If $C = 100\ \mu F,\ R_1 = R_2 = 5\ \Omega,\ R_3 = 1000\ \Omega$, obtain the $\frac{v_o}{v_i}$ in Laplace domain, when the op-amp is an ideal op-amp.

[1 Mark]

v. If the system described in (iv) above, subjected to a unit step input, obtain the $\frac{v_o}{v_i}$ in time domain.

[2 Marks]

b) Transfer function of a system is given by $G(s) = \frac{a}{s+a}$. The system was excited by a unit step input at the time $t = 0$. Calculate the time t when the system response is 0.63.

[3 Marks]

Q3. a) The mass-damper system shown in the figure Q3 is subjected to the forcing function $\hat{f}(t) = \hat{F}.\cos(\omega t)$. The displacement of mass is $\hat{x}(t)$.

i. State the forcing function in complex exponential form.

[1 Mark]

ii. Obtain the mathematical model describing motion of the mass in time domain.

[1 Mark]

iii. Explain why it is possible to use $\hat{x}(t) = Ae^{j\omega t}$ with same ω as $\hat{f}(t)$ to replace $\hat{x}(t)$ in (ii) above when the system is in steady state.

[1 Mark]

iv. Obtain an expression to describe complex displacement $\hat{x}(t)$.

[3 Marks]

v. The mechanical **impedance** is defined as $\hat{z} = \hat{f}/_{\hat{v}}$ where $\hat{v}$ is complex velocity. Obtain the mechanical **reactance**.

[3 Marks]

b) Harmonic oscillation of a system is described as superposition of the following two models.

$\hat{x}_1 = A_1 e^{j(\omega t+\varphi_1)}$, $\hat{x}_2 = A_2 e^{j(\omega t+\varphi_2)}$

i. Draw the phasor diagram to show superposition of two harmonics.

[1 Mark]

ii. Obtain the amplitude A and the phase φ of the vibration of entire system.

[2 Marks]

Q4. A permanent magnet (fixed field) DC motor is shown in figure Q4. The rotating part of the DC motor is called rotor and it carries armature circuit. The resistance of the armature circuit is R, and the inductance is L. When the motor is rotating, it produces voltage e in rotor circuit called back emf, that opposes current $i(t)$ flowing from power supply battery V to motor. The rotor has inertia of J and damping coefficient of b. The torque produced by the motor is T. Angle of rotation of motor shaft is $\theta(t)$ and angular velocity of motor shaft is $\dot{\theta}(t)$.

Assume $T(t) = ki(t)$ and $e(t) = k\dot{\theta}(t)$ where k is a constant.

Notes:

1. Inverse of Matrix A is $A^{-1} = \frac{Adjoint\ (A)}{Determinant(A)}$
2. Transpose of cofactor matrix is defined as adjoint matrix.
3. C_{ij} th element of cofactor matrix is defined as $C_{ij} = (-1)^{(i+j)} \times M_{ij}$ where, M_{ij} is the respective Minor).
4. Minor M_{ij} of 2×2 matrix can be obtained by removing i th row and j th column from 2×2 matrix.

a) Obtain the mathematical model of electrical system of the DC motor.

[2 Marks]

b) Obtain the mathematical model of mechanical system of the DC motor

[1 Mark]

c) Taking state vector as $[X_1 \quad X_2]^T = [\dot{\theta} \quad i]^T$, obtain the state space model of the system.

[2 Marks]

d) Taking $R = 1\,\Omega$, $b = 1\frac{Nm}{rad.s}$, $L = 1\,H$, $J = 1\,kgm^2$ and $k = 1$, obtain the characteristic equation using the state space model obtained above.

[2 Marks]

e) Obtain the state transition matrixes $\emptyset(s)$ and $\emptyset(t)$.

[4 Marks]

f) Based on the results of (e), state whether this system is stable or not and give reasons for your answer.

[1 Mark]

Q5. a) A non-linear system is shown in figure Q5a. u_1 constant voltage supply and u_2 is constant current supply, which are inputs to the system. The system has two 1 F capacitors and one 1 H inductor. The two resistors shown are non-linear and their behavior is as follows.

R1: The current I_1 through resistor $R1$ is function G of V_1, where $I_1 = G(V_1) = V_1^3$

R2: The voltage V_2 across resistor $R2$ is function r of I_2 where $V_2 = r(I_2)$. The function r is defined as shown in figure Q5b.

X_1 *and* X_2 are voltages and X_3 is current as shown on the figure Q5a are outputs of the system.

i. Obtain the differential equation describing $\dot{X}_1$ in terms of G, u_1, u_2, X_1 and X_3.

[2 Marks]

ii. Obtain the differential equation describing $\dot{X}_2$ in terms of X_3.

[1 Mark]

iii. Obtain the differential equation describing $\dot{X}_3$ in terms of X_1, X_2, r and X_3

[2 Marks]

iv. Assume u_1 is 1 Volt constant voltage supply and u_2 is 27 Ampere constant current supply, calculate the equilibrium state $X^0 = [X_1^0 \quad X_2^0 \quad X_3^0]^T$ for the system.

[3 Marks]

v. Obtain the linearized state space model of the system about the equilibrium point.

[3 Marks]

b) State advantages of state space system analysis over transfer function model analysis for dynamic systems.

[1 Mark]

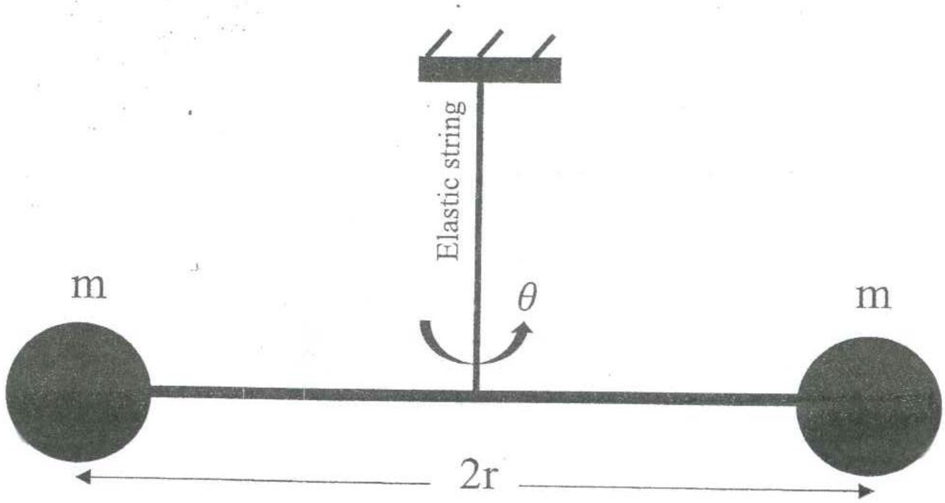

Figure Q1

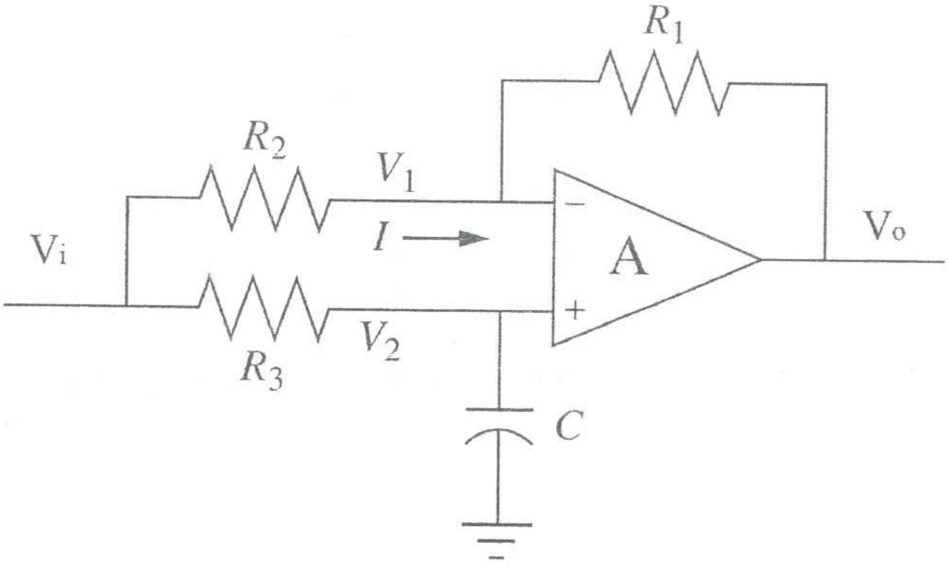

Figure Q2

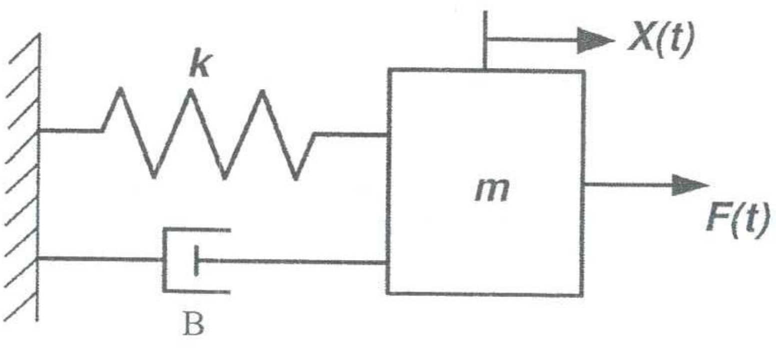

Figure Q3

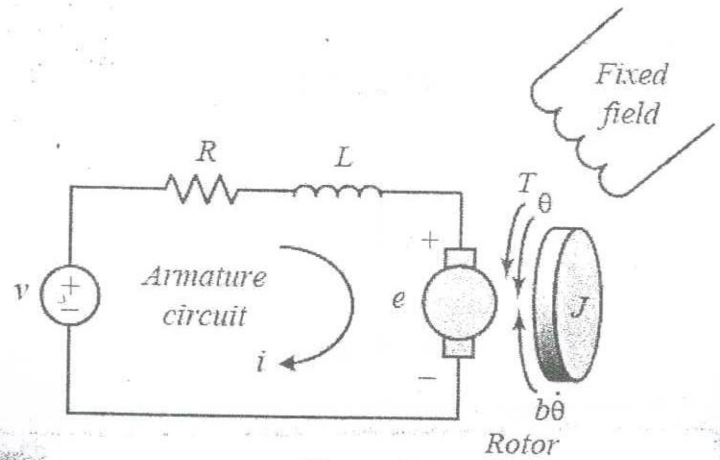

Figure Q4

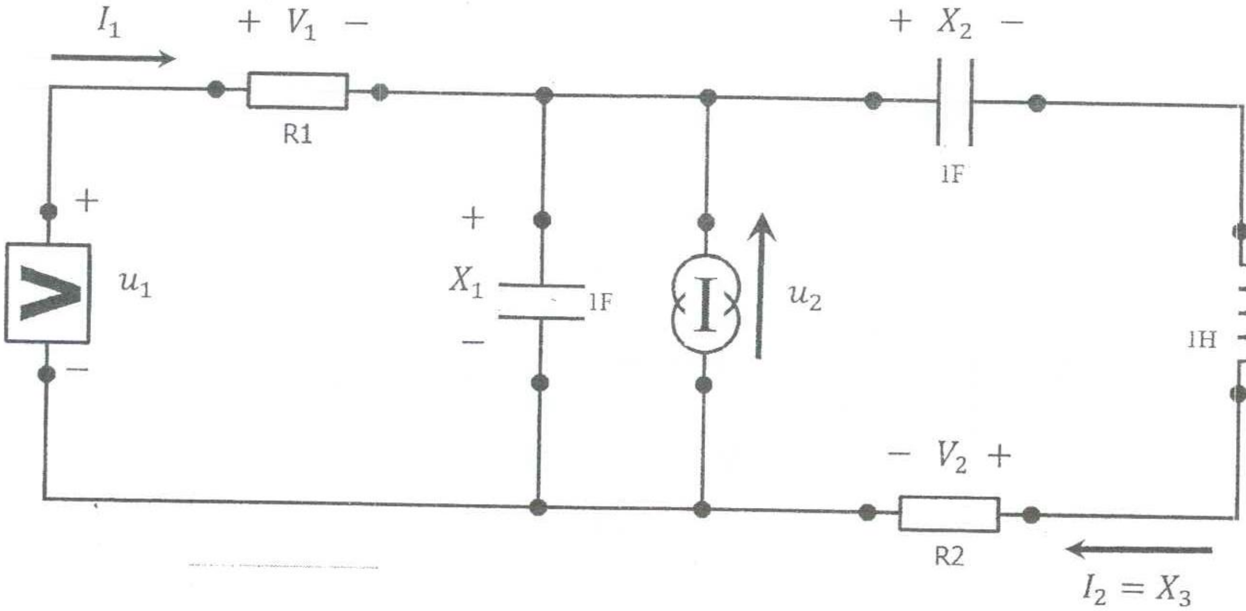

Figure Q5a

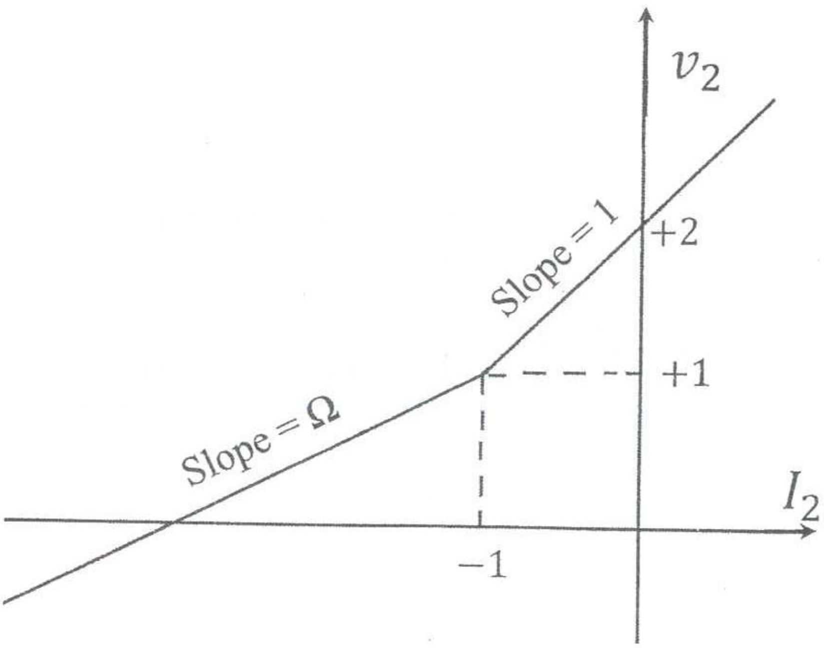

Figure Q5b – function $r(I_2)$

Laplace Transforms Table

$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2+\omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2+\omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1+at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1, 2, 3$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
t^ne^{-at}	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t) \quad Unit\ impulse$	$1 \quad all\ s$
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		