UNIVERSITY OF RUHUNA

Faculty of Engineering

Semester 5 Examination in Engineering: May 2023

Module Number: CE5253 **Module Name: Uncertainty in Engineering Measurements (C-18)**

[Three Hours]

[Answer all questions. Each question carries **FIFTEEN** marks]
All Standard Notations denote their regular meanings

Q1. a) Two different observers measured an angle using the same instrument, as indicated in Table Q1-1. Determine the following using the data available.

i. The standard deviation of each observer's readings.
ii. The standard error of the arithmetic means.
iii. The most probable value (MPV) of the angle.

[6.0 Marks]

b) Measured angles surrounding a station "O" are as given below.

a	=	75°	16'	10"	wt. 1
b	=	80°	38'	27"	wt. 2
c	=	150°	28'	15"	wt 1
d	=	53°	37'	13"	wt 1
a+b	=	155°	54'	45"	wt. 2
c+d	=	204°	05'	30"	wt. 2

Starting from fundamentals, find the most probable values of a, b, c, and d using the direct method (Method of correlatives) of least squares.

[9.0 Marks]

Q2. a) Mean traffic flow in a road is expressed as *350 ± 21* vehicles on a particular road section in a given hour. Explain the meaning of this statement.

[2.0 Marks]

b) A regression analysis yields the line $\hat{y} = 30 + 0.5x$. One of the subjects, Saman, has $x = 60$ and $y = 52$.

i. Calculate Saman's predicted value, $\hat{y}$.

[1.0 Mark]

ii. Calculate the error in Saman's estimation.

[1.0 Mark]

c) Linear regression line for traffic flow in road A is $U_s = 120\left(1 - {k}/{20}\right)$, while it is $U_s = 80\left(1 - {k}/{20}\right)$ for road B. Interpret the relationship between flow and mean speed for these two roads and discuss the characteristics of traffic flow in these two roads.

[3.0 Marks]

d) Models for these two roads have fitted with R^2 values of 0.75 and 0.3 for road A and B, respectively. Explain the meaning of these values in terms of the accuracy of the models.

[4.0 Marks]

e) One part of a given road is damaged and hence there is a difficulty in traffic movement in that part. It was observed that time to pass this damaged part is increased with the length of the vehicle. Draw the shape of a linear regression, which can depict the relationship between time and the length of the vehicle. [Explain your decision-making procedure and name the axes of your graph.]

[4.0 Marks]

Q3. The data collected during a vehicle speed distribution study is given in Table Q 3- 1. Determine the following statistical parameters related to vehicle speed referring to the Z table given in Table Q3-2 and useful equations given at the end of the paper.

a) i. Mean speed

[2.0 Marks]

ii. Standard deviation

[2.0 Marks]

iii. Standard error

[2.0 Marks]

iv. 95% confidence interval (two tail)

[2.0 Marks]

b) If $f(t) = t\exp^{(-t^2/2)}$ when $t \geq 0$ and $f(t) = 0$ when $t < 0$, compute the mean and the variance of x.

[5.0 Marks]

c) Determine the mean and the variance of a random variable x of which probability function is given by

$$f(x) = \begin{cases} 1/12 & x = 1 \\ 2/3 & x = 2 \\ 1/4 & x = 3 \end{cases}$$

[2.0 Marks]

Q4. a) Compute the Cov[x, y] and $\rho(x, y)$ for the random variables, x and y given by the joint density function

$$f(x,y) = \begin{cases} 6xy(2 - x - y) & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

[6.0 Marks]

b) The thickness of a full-depth asphalt pavement is determined by the equation

$$t = (0.25 + 0.125\log(n))\left[\sqrt{\left(\frac{75P}{\pi E}\right)^2 - a^2}\right]\sqrt[3]{E/E_p}$$

in which t is the thickness of asphalt concrete; n is the number of load repetitions; P is the wheel load; E is the modulus of the subgrade; a is the tire contact radius, and E_p is the modulus of asphalt concrete. Assume that t = 6.5in., n = 100, 000., P = 9000lb, E = 10, 000 psi, a = 6in., and E_p = 400, 000psi and that the coefficients of variation of t, n, P, E, a, and E_p are all 10%. Also assume that t has a normal distribution and that the reliability is based on the required t obtained from the above equation versus the designed t of 6.5in. Determine the reliability of the design based on Taylor series expansion considering first order terms only.

[9.0 Marks]

Tables, Figures, and Equations

Table Q1-1: Observation

Observer A			Observer B		
°	'	"	°	'	"
86	34	10	86	34	05
86	33	50	86	34	00
86	33	40	86	33	55
86	34	00	86	33	50
86	33	50	86	34	00
86	34	10	86	33	55
86	34	00	86	34	15
86	34	20	86	33	44

Table Q3-1 Speed Observation Data

Speed Class	Observed Frequency
$34 \leq X < 36$	3
$36 \leq X < 38$	5
$38 \leq X < 40$	7
$40 \leq X < 42$	5
$42 \leq X < 44$	20
$44 \leq X < 46$	30
$46 \leq X < 48$	40
$48 \leq X < 50$	89
$50 \leq X < 52$	85
$52 \leq X < 54$	40
$54 \leq X < 56$	25
$56 \leq X < 58$	18
$58 \leq X < 60$	7
$60 \leq X < 62$	3
$62 \leq X < 64$	2
$64 \leq X < 66$	1

Table Q3-2 Standard Normal Probabilities

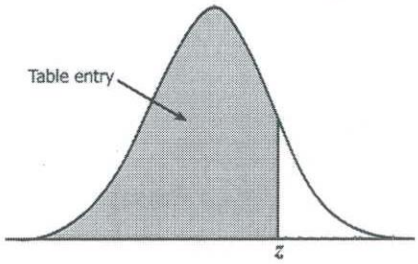

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Useful Equations

$$\bar{X} = \frac{\sum_{i=1}^{K} n_i X_i}{N}$$

$$S = \sqrt{\frac{N\sum_{i=1}^{K} n_i {X_i}^2 - (\sum_{i=1}^{K} n_i X_i)^2}{N(N-1)}}$$

$$S_M = \frac{S}{\sqrt{N}}$$

$$\bar{X} - Z_{\alpha/2} S_M < \mu < \bar{X} + Z_{\alpha/2} S_M$$

$$E[g] = g(\mu) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial^2 g}{\partial x_i \partial x_j}\right) Cov[x_i, x_j]$$

$$V[x] = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial g}{\partial x_i}\right)_{\mu}\left(\frac{\partial g}{\partial x_j}\right)_{\mu} Cov[x_i, x_j]$$