UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: May 2023

Module Number: ME 5301 **Module Name: Advanced Control Systems**

[**Three** Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:
Some necessary equations and a partial table of Laplace transformation pairs have been provided on the question paper. **You may make additional assumptions**, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

The standard form of a second-order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;

$T_s = \frac{4}{\zeta\omega_n}$ (±2% settling time);

Percentage Overshoot = $e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

Bilinear transform $s = \frac{2}{T}\frac{(Z-1)}{(Z+1)}$

Q1. A second-order system is described by the equation (1) where y is the system output and u is the input.

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 7y = 7u \qquad - (1)$$

a) Convert the equation (1) into transfer function form in the Laplace domain. (i.e., Obtain $G(S)$.)

[1 Mark]

b) What assumptions have you made during (a) above?

[1 Mark]

c) When the input is a unit step, obtain the steady-state gain and steady-state error of the system.

[1 Mark]

d) Determine the damping ratio, natural frequency, and roots of the system.

[2 Marks]

e) Sketch S-plane and plot locations of roots (not necessary to plot to scale). In the same plot, sketch the lines of constant natural frequency and constant damping ratio with clear labels.

[3 Marks]

f) Figure Q1 shows the above system $G(S)$ with a controller $C(S)$, feedback dynamics $H(S)$, and disturbance $D(S)$. Obtain the expression for $Y(S)$ in terms of $R(S)$, $C(S)$, $D(S)$, $G(S)$, and $H(S)$.

[2 Marks]

g) The system shown in Figure Q1 has $H(S) = 1$ and feels a disturbance of $\frac{2}{s}$ while subjected to a unit step input. The system has a proportional-only controller having a gain of 25. Determine the steady-state error of the closed-loop system.

[2 Marks]

Q2. a) The dynamics of two different systems are given by,

$G_1(s) = 1/(s^3 + 6s^2 + 45s)$ and

$G_2(s) = (0.075s^2 + s + 1)/(s^3 + 3s^2 + 5s)$

These systems are controlled by simple closed-loop proportional-only controllers. The feedback gain is 1. Both systems have a closed-loop pole located at $s = -1$

i. Draw the block diagram of one of the closed-loop systems.

[1 Mark]

ii. Obtain the characteristic equation corresponding to each system.

[2 Marks]

iii. Obtain roots of each system separately when the gain of the proportional-only controller is 40.

[2 Marks]

iv. Based on the roots of the given systems, draw the time domain response of each system under unit step input, in two separate figures. (Simple freehand sketches showing the shape of time domain responses are expected.) State the reasoning you used to develop each graph.

[4 Marks]

b) A control system has a single proportional gain controller. When the step response of this system is observed through simulations, the response was underdamped initially. However, after some time, the system settled to low amplitude sustained oscillations while having an average steady-state error of 0.1 units.

i. What is the method you use to eliminate the steady state error?

[1 Mark]

ii. Briefly explain the reasons for sustained oscillations.

[1 Mark]

iii. What is the technique you use to reduce sustained oscillations in the system output and how does that technique reduce sustained oscillations?

[1 Mark]

Q3. A first-order system has transfer function $G(s) = 1/(s+2)$. The plant control signal is routed through an amplifier having transfer function $G_A(s) = 1/(s+4)$.

a) Draw the block diagram of the closed-loop negative feedback system with a single feedback gain and obtain the characteristic equation.

[1 Mark]

b) The user requirements state that the maximum overshoot is 4% and system must settle down within 2.5 seconds under 2% settling time criteria. Obtain the required roots of the characteristic equation.

[3 Marks]

c) According to the requirements and results obtained in Q3.b), show that a proportional-only feedback gain is not sufficient to achieve the expected performance from the system.

[1 Mark]

d) Suggest a suitable compensator to achieve the expected performance as stated in Q3.b). Show your answer as an additional block to the Q3.a), by redrawing the block diagram with the additional block. Use variables to indicate still unknown values.

[1 Mark]

e) If the compensator used to achieve the expected performance as given in Q3.b), has zero located at $(-6, j0)$, find the location of the pole of the compensator.

[4 Marks]

f) Briefly explain the statement "The roots of the characteristic equation affect the responsiveness of system to a user input". (Maximum of 5 sentence answer is expected.)

[2 Marks]

Q4. a) A plant has the following transfer function.

$$G(s) = \frac{K}{1 + Ts}$$

i. Obtain expressions to describe the magnitude and phase of the above system at steady state when subjected to a sinusoidal input.

[2 Marks]

ii. Draw the harmonic response diagram of the system and clearly show the expressions describing values of key points.

[2 Marks]

b) A transfer function of a system is given below.

$$G(s) = \frac{s}{s^2 + 5s + 4}$$

ii. Calculate the magnitude and phase angle of the system when the system is subjected to a sinusoidal input having a frequency of 20 rad/s.

[2 Marks]

iii. Obtain the steady state response of the system when it is subjected to an input of $3\sin(20t + 10^0)$

[2 Marks]

c) The bode plot shown in Figure Q4 is obtained from a third-order system controlled by a simple proportional-only controller. Obtain the maximum value of proportional gain and corresponding cross-over frequency before closed loop system becomes unstable.

[4 Marks]

Q5. Figure Q5 shows a rotary inverted pendulum system and the corresponding free-body diagram. The motor of the system rotates the arm, and the pendulum is pivoted to the end of the arm as shown. The pivot joint is free to rotate and has negligible friction and damping. The goal of a proposed control system is to keep the pendulum upright by controlling the arm of the system. Without active control, the pendulum will not stay upright. The system uses an armature-controlled brushed permanent magnet DC motor.

a) Derive the differential equations describing the electrical and mechanical domains of the motor and convert them to the Laplace domain assuming zero initial conditions. (Note: The DC motor has an inductance L, resistance R, Inertia J, damping constant B, supply voltage v(t), shaft speed w(t), Torque T, back emf constant k_e and torque constant k_t)

[4 Marks]

b) Build a non-linear system dynamics model to simulate this rotary inverted pendulum system, using a block diagram. In your answer, include details about critical configurations, locations of the sensors, joints, and forces. State the expected function of each of the blocks used. Assume that the input to the simulator is the torque and the output is the pendulum angle at this stage.

[3 Marks]

c) Indicating the system modeled in Q5.b) using a single block, draw a suitable closed-loop control system using a block diagram to keep the pendulum at the upright position. Clearly indicate the controller you are using with reasons.

[1 Mark]

d) The motor driver of DC motor has a current limit of ± 5 A and a dead band of ± 0.1 A. The DC motor accepts current as input and outputs torque. Include these details and redraw an improved model.

[2 Marks]

e) Suggest a suitable sensor to measure the angle of the pendulum with sufficient accuracy and precision. If you are to implement this system as a physical prototype, what is the expected accuracy?.

[2 Marks]

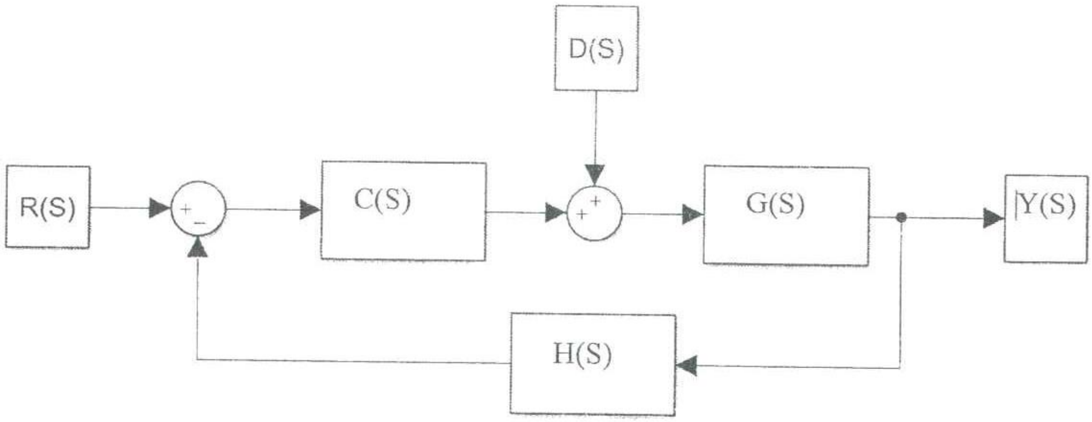

Figure Q1

(Figure Q4 is printed on next page.)

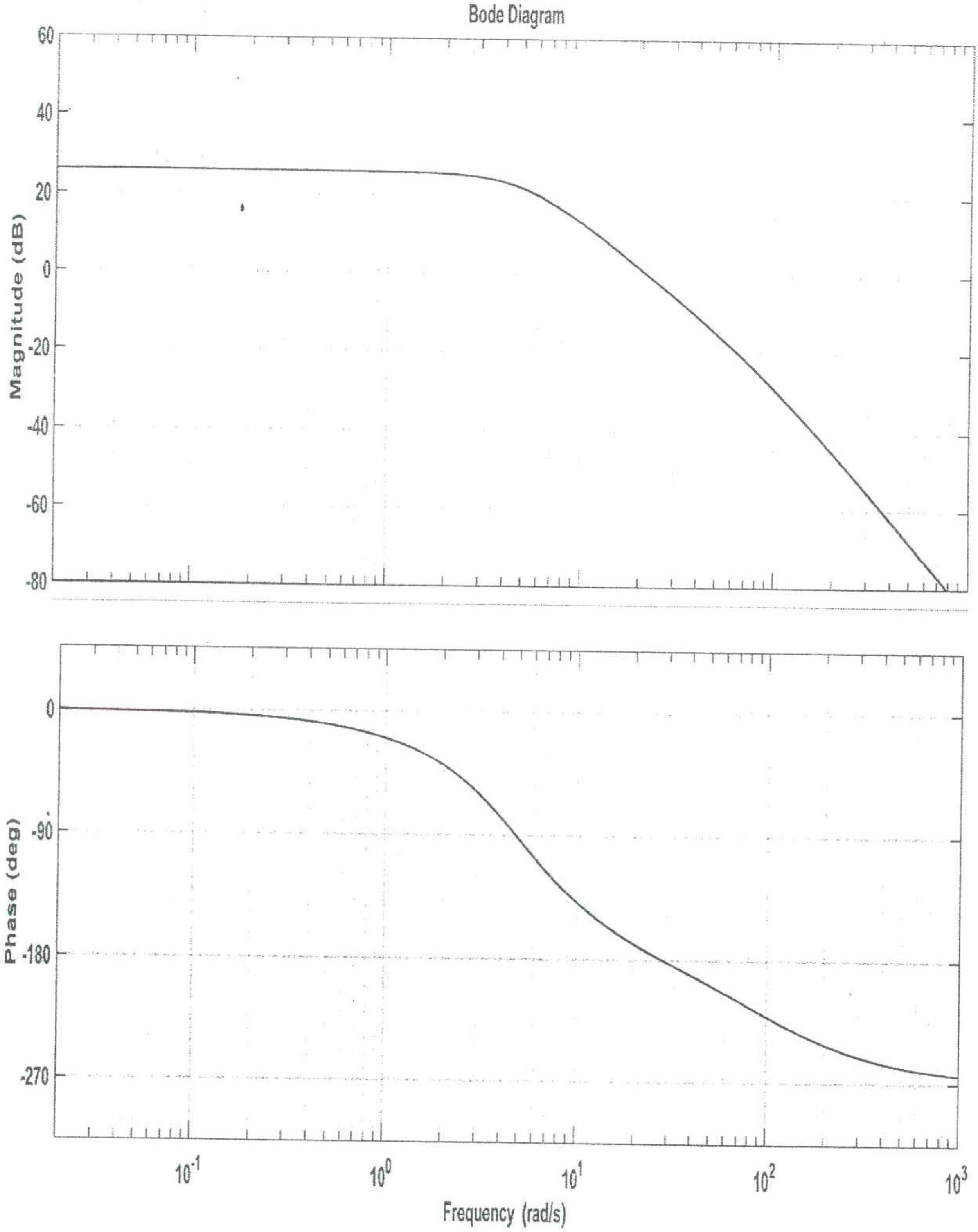
Bode Diagram
Magnitude (dB)
60
40
20
0
-20
-40
-60
-80
Phase (deg)
0
-90
-180
-270
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

Figure Q4

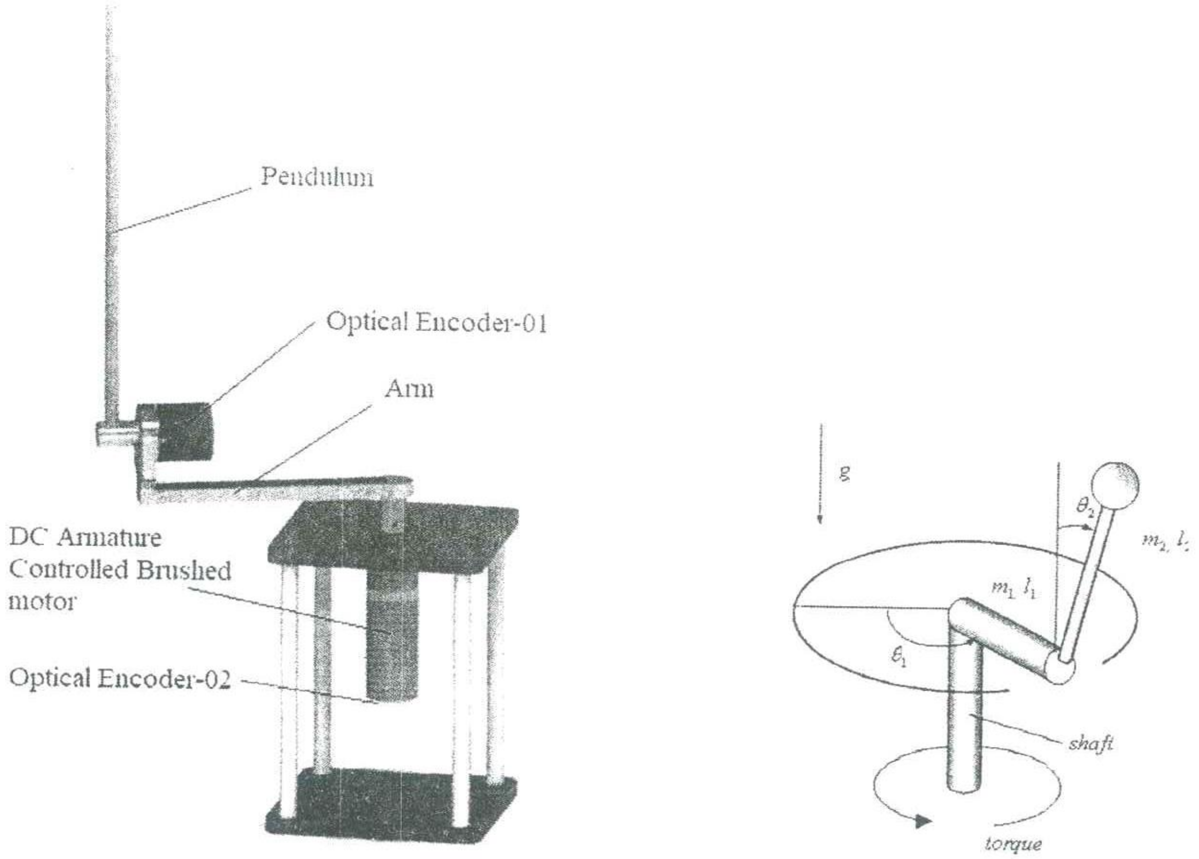
Pendulum
Optical Encoder-01
Arm
DC Armature Controlled Brushed motor
Optical Encoder-02
g
θ2
m2, l2
m1 l1
θ1
shaft
torque

Figure Q5

Laplace Transforms Table

$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$
$\frac{1}{2} t^2 e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 + \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 + \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1 + at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1, 2, 3$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s} F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t) dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *Unit impulse*	1 *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2 f}{df^2}$	$s^2 F(s) - sf(0) - f'(0)$
$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - f^{n-1}(0)$		