UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: May 2023

Module Number: EE7205 **Module Name: Digital Signal Processing (C-18)**

[Three Hours]

[Answer **all questions,** each question carries **10 marks]**

Notes:

- **All notations have their usual meaning.**

Q1 a) Consider a discrete-time system with the difference equation,

$$y[n] = 0.5y[n-1] + x[n] + 2x[n-1].$$

Find the output response y[n] for an input sequence $x[n] = \{\underline{1}, 2, 3, 4, 5\}$ with initial condition y[-1] = 0. State any assumptions you make.

[2.0 Marks]

b) Consider a discrete-time system with the difference equation,

$$y[n] + 0.8y[n-1] + 0.2y[n-2] = x[n] + 0.5x[n-1].$$

Find the transfer function H(z) of the system by evaluating its Z-transformation.

[2.5 Marks]

c) i) Define what is meant by BIBO stability for a discrete-time system.

ii) What does it imply about the system's behavior in response to bounded input signals?

iii) Explain why BIBO stability is a desirable property for practical systems.

[3.0 Marks]

d) Consider a discrete-time system with the transfer function given below. Determine the stability of this system based on its transfer function. Explain your answer mentioning the approach to determine the stability of a given system using its transfer function.

$$H(z) = \frac{1}{(1 - 1.5z^{-1} + 0.7z^{-2})}$$

[2.5 Marks]

Q2 a) Implement a cascade structure for the following IIR filter transfer function.

$$H(z) = \frac{1 + 0.5z^{-1} + 0.2z^{-2}}{1 - 0.8z^{-1} + 0.4z^{-2}} \cdot \frac{1 + 0.3z^{-1}}{1 - 0.6z^{-1} + 0.2z^{-3}}$$

Hint: Use direct from II structures to represent the cascade blocks.

[4.0 Marks]

b) The Figure Q2.b shows a 2-stage lattice filter with the output y(n) given by, $y(n) = x(n) + \alpha_2(1)x(n-1) + \alpha_2(2)x(n-2)$. Derive expressions for the reflection coefficients K_1 and K_2 in terms of α_2.

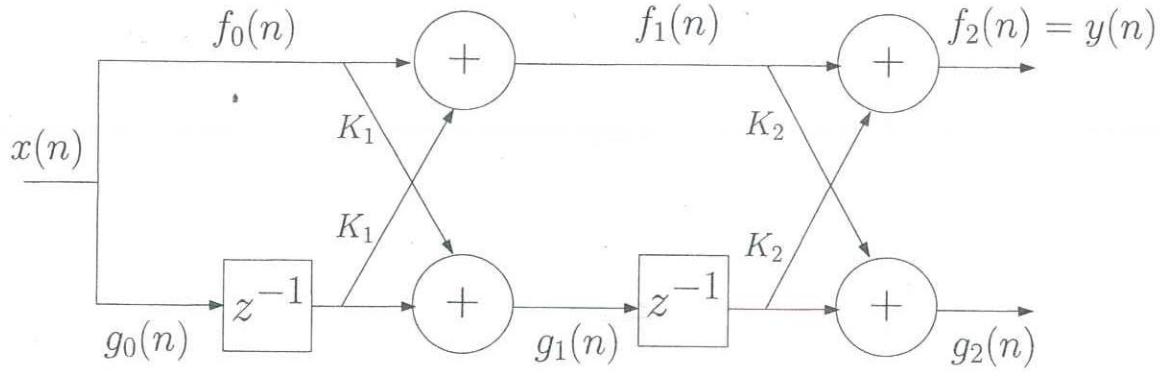

Figure Q2.b

[3.0 Marks]

c) Implement the filter represented by following transfer function in both direct form I and direct form II.

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.2z^{-1} - 0.2z^{-2}}$$

[3.0 Marks]

Q3 a) Explain the significance of the Discrete Fourier Series (DFS) in analyzing discrete-time periodic signals. How does it aid in understanding the frequency components of a signal? Provide an example to illustrate your explanation.

[2.0 Marks]

b) Discuss the Gibbs phenomenon in relation to the Discrete-Time Fourier Transform (DTFT) and its effect on reconstructing signals. Provide an example to support your explanation.

[2.0 Marks]

c) Determine the DTFT of the following sequences. $u[n]$ denotes the unit step function.

i) $x[n] = 0.5^n u[n]$
ii) $x[n] = 2^n\ u[-n]$

[3.0 Marks]

d) Compute the Discrete Fourier Transform (DFT) of the following sequences.

i) $x = [1, 0, -1, 0]$
ii) $x = [j, 0, j, 1]$
iii) $x = [1, 1, 1, 1, 1, 1, 1, 1]$

[3.0 Marks]

Q4 a) How does the Radix-2 Decimation-In-Time (DIT) Fast Fourier Transform (FFT) algorithm enhance computational efficiency in the spectral analysis compared to the direct computation of the Discrete Fourier Transform (DFT)?

[3.0 Marks]

b) Split the N-point DFT

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn}, \quad k = 0,1,2, \dots, N-1$$

into **two** summations such that

$$X[k] = G[k] + W_N^k H[k]$$

where $G[k]$ denote the sum over the even-numbered discrete-time indices $n = [0, 2, 4, \dots, N-2]$ and $H[k]$ denote the sum over the odd-numbered indices $n = [1, 3, 5, \dots, N-1]$. Note that the twiddle factor $W_N = e^{-\frac{j2\pi}{N}}$ and $W_N^{2nk} = W_{N/2}^{kn}$.

[3.0 Marks]

c) Using the DIT Radix-2 FFT algorithm, determine the DFT of the sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$.

[4.0 Marks]

Q5 a) Briefly compare FIR and IIR filters in terms of the following parameters.

i) Design complexity
ii) Accuracy
iii) Stability
iv) Memory requirements
v) Phase response

[2.5 Marks]

b) i) Convert the analog filter with following system function into a digital IIR filter by means of impulse invariance method.

$$H_a(s) = \frac{(s + 0.1)}{(s + 0.1)^2 + 9}$$

ii) Comment on the stability of the filter by observing the pole locations of the analog filter. Justify your answer.

iii) Is the above impulse invariance approach suitable for designing high pass filters? Explain your answer.

[5.0 Marks]

c) How does the choice of windowing function affect the passband/stopband ripples and the smooth transition from passband to stopband? Explain your answer by considering Hamming and Rectangular windowing functions as examples.

[2.5 Marks]