UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: May 2023

Module Number: CE7202 **Module Name: Computer Analysis of Structures**

[Three Hour]

[Answer all questions. Marks for each question carries as indicated]

Q1. An idealized steel truss structure is shown in Fig. Q1. For the design purpose, the structural engineer wants to determine the reaction forces at the supports and the member forces in all the elements when the maximum load of 20 kN is applied at the node 'C'. Assume that the cross-sectional area, A, for all the elements is 10 cm^2 and the elastic modulus, E, for steel is 200 GPa.

A) For the given truss structure, determine the degree of statical indeterminacy, SI°.

[2.0 Marks]

b) Using matrix flexibility method, determine the member forces in all the element in the truss structure. With usual notations, member flexibility matrix for a truss element is [L/AE].

[8.0 Marks]

c) Determine the vertical deflection at node 'C' in millimetres.

[5.0 Marks]

Q2. A two-dimensional frame structure is shown in Fig. Q2. It has been found that the support A has been settled by 4mm downward and a point load of 12 kN has been applied at node B. Member lengths are as indicated in the Fig. Q2.

a) Matrix stiffness method is governed by degree of kinetic indeterminacy (KI°). Explain briefly what is kinetic indeterminacy and determine KI° for the plane structure shown in Fig. Q2.

[3.0 Marks]

b) Using matrix stiffness method, determine the support reactions and the nodal deformations (i.e., displacement and rotation) at joint B. All the members have flexural rigidity (EI) of 10^5 kNm^2.

[12.0 Marks]

Use stiffness matrix for a two-dimensional beam element with negligible axial deformation as,

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Q3. a) i.) List two advantages of modelling a real world problem using finite element analysis.

ii.) Mention the places where it is necessary to place a node, during discretization of a model in finite element analysis.

[2 Marks]

b) Using stiffness equation for 3D continua;

$$\left[K^{n}\right] = \int^{v} [B]^{T} [D][B] d(vol)$$

Show that element stiffness for one dimensional bar element is given by

$$\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$ where, E, A, and L have their usual meanings.

[3 Marks]

c) Determine the displacements and support reactions for the bar combination shown in Fig. Q2. Bar **AB** is made of Aluminium with Young's modulus $E_a = 80 \times 10^9$ N/m² and cross section $A_a = 750$ mm². Bar BC is made of steel with Young's modulus $E_s = 200 \times 10^9$ N/m² and cross section $A_s = 1200$ mm². Horizontal load of P = 250 kN is applied at point B.

[5 Marks]

Q4. Pin-jointed 2D truss is pinned supported at Node **A** and roller supported at Nodes B and **C** as shown in Fig. Q4. The Young's modulus $E = 200$ GPa for all three elements and cross-section area $A = 5\times10^{-4}$ m² for elements (1) and (2), $5\sqrt{2}\times10^{-4}$ m² for element (3). Truss system is subjected to a horizontal force of 1500 kN at Node **C**, as shown in Fig. Q4.

a) Find the element stiffness matrix of the 3 elements with respect to a selected global coordinate system.

[3 Marks]

b) Determine the global stiffness matrix of the system.

[1 Mark]

c) Define the boundary condition and loading condition for each node.

[2 Marks]

d) Determine the displacements at Nodes **B** and C.

[2 Marks]

e) Determine the support reactions at each node.

[2 Marks]

(Use the stiffness matrix for a 2D-bar element as shown below.)

$$\left[k^{e}\right] = \frac{EA}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

where $c = Cos\Theta$, $s = Sin\Theta$ and Θ is the anticlockwise angle at node measured from the global X-axis to the local x-axis of the bar element.

Q5. A beam subjected to different loading is shown in Fig. Q5 and is supposed to be analyzed using the finite element technique with three elements as shown. The Young's Modulus of the beam is E and the second moment of area is I and $EI = 20\times10^6$ N/m².

a) Find the equivalent nodal forces and moments at the four nodes of the beam and draw the resultant on the model.

[2 Marks]

b) Determine the Nodal deformations and reactions.

[7 Marks]

c) Propose a possible method to increase the accuracy of the answer in the analysis.

[1 Mark]

(Ignore the axial effect and use the stiffness matrix for a beam element as shown below.)

$$[k^e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

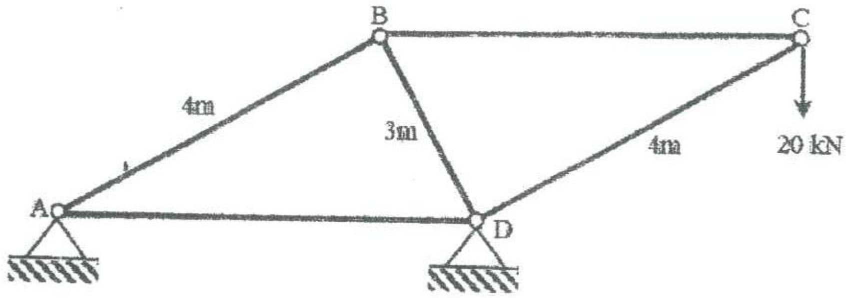


Fig. Q1

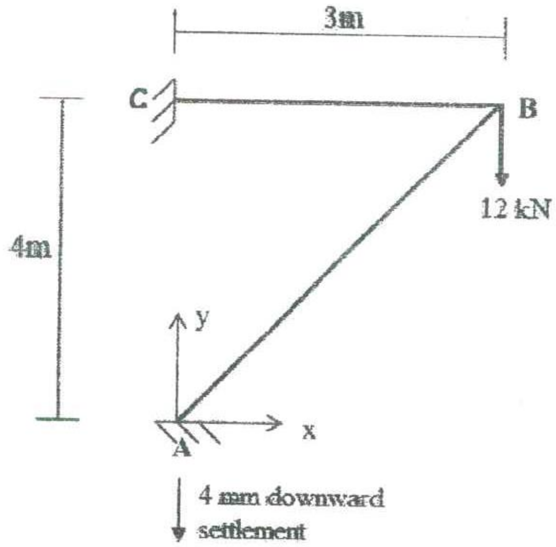


Fig. Q2

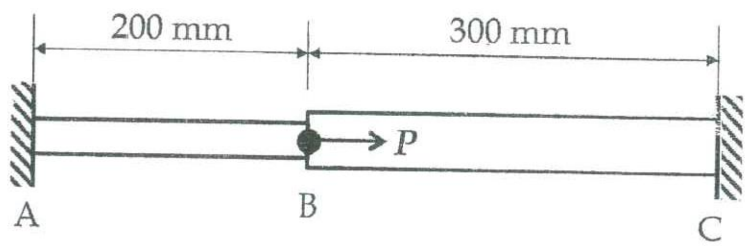
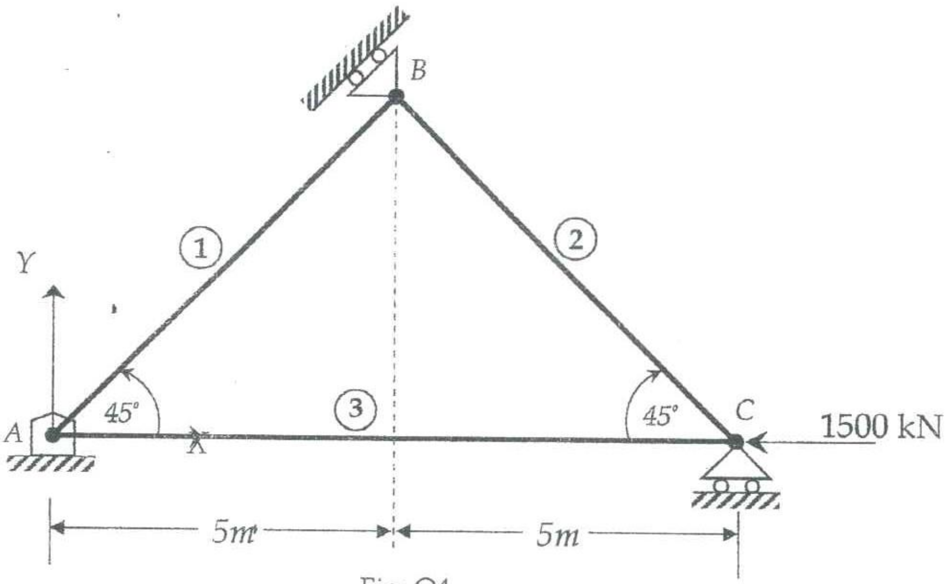
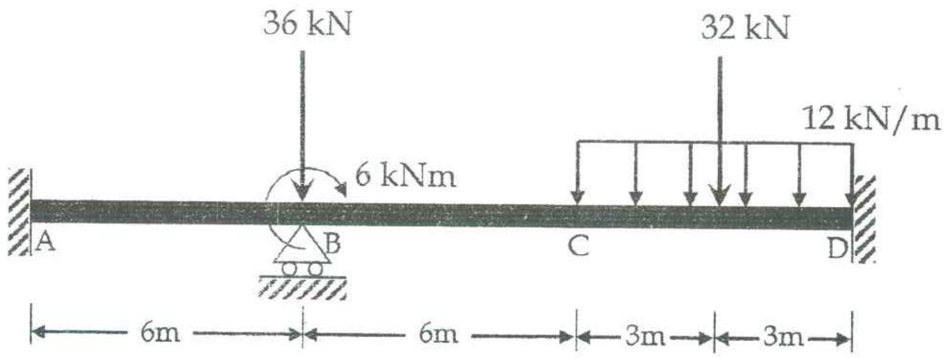


Fig. Q3



B
1
2
Y
45°
3
45°
C
1500 kN
A
X
5m
5m

Fig. Q4



36 kN
32 kN
12 kN/m
6 kNm
A
B
C
D
6m
6m
3m
3m

Fig. Q5