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UNIVERSITY OF RUHUNA - FACULTY OF ALLIED HEALTH SCIENCES

DEPARTMENT OF PHARMACY

FIRST BPHARM PART I EXAMINATION - DECEMBER 2023

PH1152 : MATHEMATICS - SEQ

TIME: TWO HOURS

INSTRUCTIONS

- There are **four** questions in this paper.
- Answer **all** questions.
- Calculators will be provided.
- No paper should be removed from the examination hall.
- Do not use any correction fluid.
- Use illustrations where necessary.

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1. a) Find the following limits:

   (i) $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$ **[15]**

   (ii) $\lim_{x\to -4} \dfrac{\sqrt{x^2+9}-5}{x+4}$ **[15]**

   b) Differentiate the function $y = x^4$ with respect to x using the **first principles**. **[25]**

   c) Suppose a large tank contains 8 Kg of a chemical dissolved in 50 liters (ℓ) of water. A tap is opened and water is added to the tank at a rate of 5 ℓ per minute. The concentration would be measured as Kg of chemical per liter of water, Kg/ℓ. Assume that the number of Kg of chemical remains constant at 8 Kg.

   (i) What is the total volume of water in the tank after t minutes? **[05]**

   (ii) What is the concentration $C(t)$ of the chemical in the tank after t minutes? **[10]**

   (iii) What is the **rate of change of concentration** after time t? **[15]**

   (iv) Determine the **rate of change of concentration** after 4 minutes and interpret your result. **[15]**

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2. a) The curve $y = ax^2 + bx + c$ passes through the point $(0,-8)$ and has a stationary point at $(-1,-5)$. Calculate the values of a, b, c. **[30]**

   b) Consider the function $f(x) = \frac{2}{3}x^3 + \frac{3}{2}x^2 - 2x + 4$.

   (i) Find the stationary points of the function $f(x)$. **[45]**

   (ii) Discuss the nature of the above stationary points using the second derivative $f''(x)$. **[25]**

3. a) Consider the function $f(x,y) = \cos 3x \sin 4y$.

   (i) Find the first partial derivatives $\dfrac{\partial f(x,y)}{\partial x}$ and $\dfrac{\partial f(x,y)}{\partial y}$ at the point $\left(\dfrac{\pi}{12}, \dfrac{\pi}{6}\right)$. [20]

   (ii) Show that the total differential df of f at the point $\left(\dfrac{\pi}{12}, \dfrac{\pi}{6}\right)$ is given by

$$df = \frac{-3\sqrt{6}}{4}\,dx - \sqrt{2}\,dy.$$

[10]

b) Given $g(x,y) = \ln(x^2+y^2)$, show that

$$\frac{\partial^2 g(x,y)}{\partial x^2} + \frac{\partial^2 g(x,y)}{\partial y^2} = 0.$$

[30]

c) Consider the function $h(x,y) = \sqrt{x^2+y^2}$.

   (i) Is $h(x,y)$ homogeneous? What is the degree of homogenity? [15]

   (ii) Prove that $h(x,y)$ satisfies the Euler's theorem. [25]

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4. a) Using the substitution $u = 2 + x^4$, integrate

$$\int x^3(2+x^4)^5\,dx.$$

[20]

b) Use the method of integration by parts to evaluate

$$\int_1^2 x^2 \ln x\,dx.$$

[30]

c) Evaluate the integral

$$\int_0^1 \frac{x-1}{x^2+3x+2}\,dx$$

using the method of partial fractions. [30]

d) Test the differential equation

$$(y^2\cos x - \sin x)\,dx + (2y\sin x + 2)\,dy = 0$$

for exactness. If it is exact, then find its solution. [20]

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