UNIVERSITY OF RUHUNA

2018/2019 Academic Year B. A. (Special) Degree - 2000 Level

2^{nd} Semester Examination – March – August, 2020

STS22613 – Inferential Statistics (English Medium)

*Answer any **four (04)** questions.*

Use of calculators are allowed.

Time: 03 Hours

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1)

a) Describe the following technical terms in the process of statistical inference.

(i). Population.

(ii). Representative sampling.

(04 Marks)

b) Give the real-life examples for each of the following situations;

(i). Interval estimation for a proportion.

(ii). Hypothesis testing of mean difference.

(05 Marks)

c) Describe the following terms used in inferential statistics with examples.

(i). Parameter.

(ii). Sample Statistic.

(06 Marks)

2) A research student has been taken 36 random samples of size two from a population with mean 3.5 and variance of 2.92. The samples are given in the table below and use this information to answer the questions.

a) Construct the sampling distribution of sample mean and present it in the form of a frequency distribution.

(07 Marks)

b) Find the **mean** and **variance** of the sampling distribution.

(08 Marks)

Sample #	Sample values	Sample #	Sample values	Sample #	Sample Values
1	1, 1	2	1, 2	3	1, 3
4	1, 4	5	1, 5	6	1, 6
7	2, 1	8	2, 2	9	2, 3
10	2,4	11	2, 5	12	2, 6
13	3, 1	14	3, 2	15	3, 3
16	3, 4	17	3, 5	18	3, 6
19	4, 1	20	4, 2	21	4, 3
22	4, 4	23	4, 5	24	4, 6
25	5, 1	26	5, 2	27	5, 3
28	5, 4	29	5, 5	30	5, 6
31	6, 1	32	6, 2	33	6, 3
34	6, 4	35	6, 5	36	6, 6

3)

a) Briefly explain the concept of "Central Limit Theorem" used in inferential statistics.

(02 Marks)

b) The foreman of a bottling plant has observed that the amount of soda in each 32-ounce bottle is actually a normally distributed random variable, with mean of 32.2 ounces and the standard deviation of .3 ounce. If a customer buys a pack of four bottles, what is the probability that the mean amount of the four bottles will be greater than 32 ounces?

(05 Marks)

c) What are the main assumptions for constructing a sampling distribution of difference of two means?

(03 Marks)

d) An electrical device made by Company A has an average life of 2500 hours, with a standard deviation of 500 hours. Another Company, B, makes this appliance with an average life of 2300 hours, and a standard deviation of 800 hours. We randomly take 300 devices from Company A and 200 devices from Company B. Calculate the probability that the average life of the sample from Company A is not 100 hours more than the average life of the sample from Company B.

(05 Marks)

4)

a) An extract of some of the Minitab dialog boxes related to inferential statistical procedures are given below. State the conditions we need to use each of the dialog boxes under given heading.

(i). Data structure.

(ii). The practical inferential situation.

(iii). The main use of dialog box.

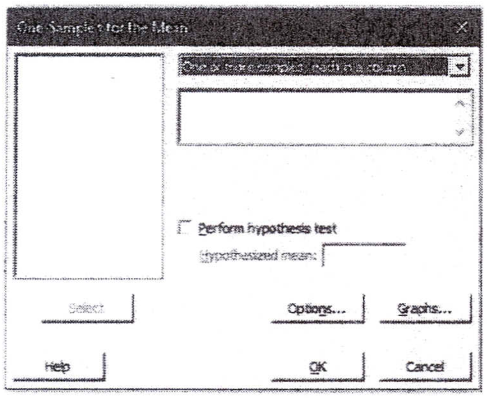

Figure 1

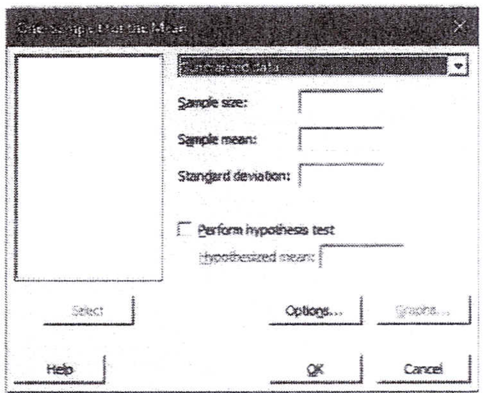

Figure 2

(06 Marks)

b) Following is an output extracted from a data analysis using Mintab software. Use the output to answer the given questions.

(i). Write down the command path to obtain the given output.

(1.5 Marks)

(ii). What is the point estimate of difference of two means?

(1.5 Marks)

(iii). Write down the standard error of the sampling distribution used in the analysis.

(01 Mark)

(iv). Is the statistical test of hypothesis significant at 5% level? Explain.

(02 Marks)

(v). Can you justify your conclusion in (iv) by using 95% confidence intervals?

(03 Marks)

Two – Sample T – Test and CI

Method

μ_1: mean of sample 1

μ_2: mean of sample 2

Difference $\mu_1 - \mu_2$

Equal variances are assumed for this analysis

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Sample 1	24	45.30	9.40	1.9
Sample 2	32	54.20	12.50	2.2

Estimation for difference

Difference	Pooled StDev	95% CI for Difference
-8.90	11.28	(15.01, -2.79)

Test

Null hypothesis $H_o: \mu_1 - \mu_2 = 0$

Alternative hypothesis $H_1: \mu_1 - \mu_2 = 0$

T-Value	DF	P-Value
-2.92	54	0.005

5)

a) What is the definition of an unbiased estimator?

(03 Marks)

b) $X_{11}, X_{12}, \ldots \ldots \ldots X_{1n1}$ and $X_{21}, X_{22}, \ldots \ldots \ldots X_{2n2}$ two random samples of size $n_1 = 25$ and $n_2 = 35$ respectively from two independent populations. Show that the difference of sample means, $\bar{X}_1 - \bar{X}_2$ is an unbiased estimator for the difference of two population means, $\mu_1 - \mu_2$.

(12 Marks)

6)

a) Why is an interval estimate better than a point estimate?

(03 Marks)

b) A random sample of 20 tax payers of UNP-controlled local authorities shows that they spend an average of Rs. 175 on administration with a standard deviation of Rs. 25. A similar survey of 15 taxpayers of JVP-controlled authorities finds an average figure of Rs. 158 with standard deviation of Rs. 30. The sample information available is given below.

$$\bar{X}_1 = 175,\ \bar{X}_2 = 158, S_1 = 25,\ S_2 = 30,\ n_1 = 20, n_2 = 15$$

Historical data provides that variances of two populations are approximately equal. You are required to estimate the true difference in expenditure between UNP and JVP controlled authorities under 95% level of confidence. Use this information to answer following questions.

a) What is the relevant statistic and its value for this situation?

(02 Marks)

b) What are the assumptions you need to consider for finding interval estimates for true mean difference?

(03 Marks)

c) Construct the 95% confidence intervals for true mean difference

(05 Marks)

d) Interpret your results in part (c).

(02 Marks)

7) Write notes on the following topics providing relevant examples.

a) Errors in hypothesis testing.

(05 Marks)

b) Test statistic in hypothesis testing.

(05 Marks)

c) Bootstrapped confidence intervals

(05 Marks)

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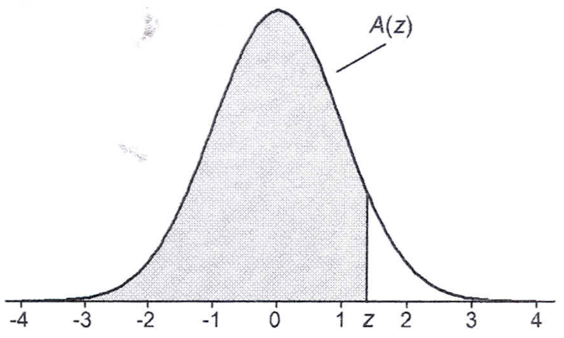
TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	Area in Two Tails				
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
1	**63.657**	31.821	**12.706**	6.314	3.078
2	**9.925**	6.965	**4.303**	2.920	1.886
3	**5.841**	4.541	**3.182**	2.353	1.638
4	**4.604**	3.747	**2.776**	2.132	1.533
5	**4.032**	3.365	**2.571**	2.015	1.476
6	**3.707**	3.143	**2.447**	1.943	1.440
7	**3.499**	2.998	**2.365**	1.895	1.415
8	**3.355**	2.896	**2.306**	1.860	1.397
9	**3.250**	2.821	**2.262**	1.833	1.383
10	**3.169**	2.764	**2.228**	1.812	1.372
11	**3.106**	2.718	**2.201**	1.796	1.363
12	**3.055**	2.681	**2.179**	1.782	1.356
13	**3.012**	2.650	**2.160**	1.771	1.350
14	**2.977**	2.624	**2.145**	1.761	1.345
15	**2.947**	2.602	**2.131**	1.753	1.341
16	**2.921**	2.583	**2.120**	1.746	1.337
17	**2.898**	2.567	**2.110**	1.740	1.333
18	**2.878**	2.552	**2.101**	1.734	1.330
19	**2.861**	2.539	**2.093**	1.729	1.328
20	**2.845**	2.528	**2.086**	1.725	1.325
21	**2.831**	2.518	**2.080**	1.721	1.323
22	**2.819**	2.508	**2.074**	1.717	1.321
23	**2.807**	2.500	**2.069**	1.714	1.319
24	**2.797**	2.492	**2.064**	1.711	1.318
25	**2.787**	2.485	**2.060**	1.708	1.316
26	**2.779**	2.479	**2.056**	1.706	1.315
27	**2.771**	2.473	**2.052**	1.703	1.314
28	**2.763**	2.467	**2.048**	1.701	1.313
29	**2.756**	2.462	**2.045**	1.699	1.311
30	**2.750**	2.457	**2.042**	1.697	1.310
31	**2.744**	2.453	**2.040**	1.696	1.309
32	**2.738**	2.449	**2.037**	1.694	1.309
34	**2.728**	2.441	**2.032**	1.691	1.307
36	**2.719**	2.434	**2.028**	1.688	1.306
38	**2.712**	2.429	**2.024**	1.686	1.304
40	**2.704**	2.423	**2.021**	1.684	1.303
45	**2.690**	2.412	**2.014**	1.679	1.301
50	**2.678**	2.403	**2.009**	1.676	1.299
55	**2.668**	2.396	**2.004**	1.673	1.297
60	**2.660**	2.390	**2.000**	1.671	1.296
65	**2.654**	2.385	**1.997**	1.669	1.295
70	**2.648**	2.381	**1.994**	1.667	1.294
75	**2.643**	2.377	**1.992**	1.665	1.293
80	**2.639**	2.374	**1.990**	1.664	1.292
90	**2.632**	2.368	**1.987**	1.662	1.291
100	**2.626**	2.364	**1.984**	1.660	1.290
200	**2.601**	2.345	**1.972**	1.653	1.286
300	**2.592**	2.339	**1.968**	1.650	1.284
400	**2.588**	2.336	**1.966**	1.649	1.284
500	**2.586**	2.334	**1.965**	1.648	1.283
750	**2.582**	2.331	**1.963**	1.647	1.283
1000	**2.581**	2.330	**1.962**	1.646	1.282
2000	**2.578**	2.328	**1.961**	1.646	1.282
Large	**2.576**	2.326	**1.960**	1.645	1.282