UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: November 2022

Module Number: EE6302 **Module Name: Control System Design (C-18)**

[Three Hours]

[Answer **all questions**, each question carries 12 marks]
Note: Formulas you may require are given in page 7. A table of Laplace transforms is attached in page 8.

Q1 a) i) Using a block diagram, show the components of a closed-loop control system.

ii) Briefly explain the purpose of the controller, the actuator and the sensor in a closed-loop control system?

iii) What is the main disadvantage of an open-loop control system?

[3 Marks]

b) i) Drawing a suitable time response, explain the terms; rise time, settling time, maximum overshoot and peak time, associated with a control system.

ii) The closed-loop system shown in Figure Q1(b) should be designed so that the overshoot does not exceed 25% and the peak time does not exceed 2 seconds.

I) Show the allowable regions in the s-plane for the poles of the closed-loop system.

II) In order to achieve maximum allowable overshoot and peak time, determine the values of p and q of the system.

[5 Marks]

c) i) Consider the system shown in Figure Q1(c1). Show that a non-zero steady-state error exists in the system for a unit-ramp input.

ii) In order to eliminate the steady-state error for a unit-ramp input, an input filter is added to the system as shown in Figure Q1(c2). Determine the input filter transfer function H(s).

[4 Marks]

Q2 a) i) In terms of s-plane point of view, what is the necessary and sufficient condition to be fulfilled to have a stable system?

ii) State the Routh's necessary and sufficient condition to have a stable system.

[1.5 Marks]

b) Using Routh's stability criterion, find the range of values to be fulfilled for the PI (Proportional-Integral) controller gains K and K_I so that the system in Figure Q2(b) to be stable. In the (K_I, K) plane, graphically show the allowable region for K and K_I.

[4 Marks]

c) i) Write the general form of matrix equations so that a system is represented in state-variable form. Name the matrices in your matrix equations.

ii) Consider the RLC circuit shown in Figure Q2(c). Writing differential equations, represent the system in state-variable form. Take the state vector x as

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where $x_1 = v_C(t)$, voltage across the capacitor and $x_2 = i_L(t)$, current through the inductor.

Input of the system is $v(t)$, the input voltage of the circuit. The output of the system is $i_R(t)$, the current through the resistor.

iii) Using the state-variable form identified in part ii), derive the transfer function of the system.

[6.5 Marks]

Q3 a) i) State the definition of the Root locus.

ii) Briefly explain the importance of the root locus.

iii) Consider the closed-loop system shown in Figure Q3(a). The transfer function $G(s) = \frac{N(s)}{D(s)}$. Prove that the zeros of the closed-loop transfer function equal to the zeros of open-loop transfer function.

[5.0 Marks]

b) Figure Q3(b) gives the root locus of the closed-loop system shown in Figure Q3(a).

i) Derive the transfer function $G(s)$ of the system.

ii) Determine whether the point $s = -2 + 3.75j$ is on the root locus.

iii) Calculate the imaginary axis crossings of the root locus.

iv) Calculate the angles of departures at open-loop poles $s = -2.5 + 0.866j$, $-2.5 - 0.866j$.

v) Find the range of gain K where the closed-loop system is stable.

vi) Sketch the expected response from the closed-loop system when the gain K is 2000. The input is a unit step.

[7.0 Marks]

Q4 a) i) What are the three basic modes of control action? State the relationship between the error (controller input) and the controller output for the above modes.

ii) Briefly explain how to find the gain required to yield a certain percent overshoot using the root locus.

iii) Briefly explain the effect of increasing the open-loop gain on the closed-loop transient response.

[4.0 Marks]

b) **Answer this question using your knowledge on root locus design technique.**

However, it is **NOT** necessary to create an accurate plot of the root locus of the given system.

Consider the closed-loop system shown in Figure Q4. The transfer function of the system is

$$G(s) = \frac{1}{(s+1)(s+2)(s+12)}$$

The input reference is a unit step input. The expected characteristics of the system response are as follow.

Percent overshoot ≈ 20%
Peak time ≈ 0.5 s

i) Design a PD controller to obtain the desired response.

ii) Calculate the steady state error of the compensated system with the PD compensator.

iii) How to eliminate the steady state error of the compensated system, designed in part i)?

[8.0 Marks]

Q5 a) Draw the asymptotic approximations of the bode plots of the following system.

$$G(s) = \frac{145s}{(s+5)(s+10)(s+25)}$$

[3.0 Marks]

b) **Answer this question using your knowledge on frequency response design technique.** However, it is **NOT** necessary to create an accurate plot of the bode diagram.

Consider the closed-loop system given in Figure Q4. The transfer function

$$G(s) = \frac{K}{(s^2+3s+7)}$$

The input R(s) is a unit step input.
The desired characteristics of the closed-loop transient response are as follows.

Percent overshoot ≈ 20%
Steady state error = 1%
Settling time < 0.4 s

A lag compensator can be designed to achieve the desired percent overshoot and the steady state error. However, the closed-loop response is slow, hence the desired settling time requirement cannot be met. Therefore, lead compensator is appropriate for the given requirements.

i) Calculate the gain required to meet 1% steady state error.

ii) Calculate the phase margin required to yield 20% overshoot.

iii) Design a lead compensator to meet the given closed-loop transient response characteristics.

[9.0 Marks]

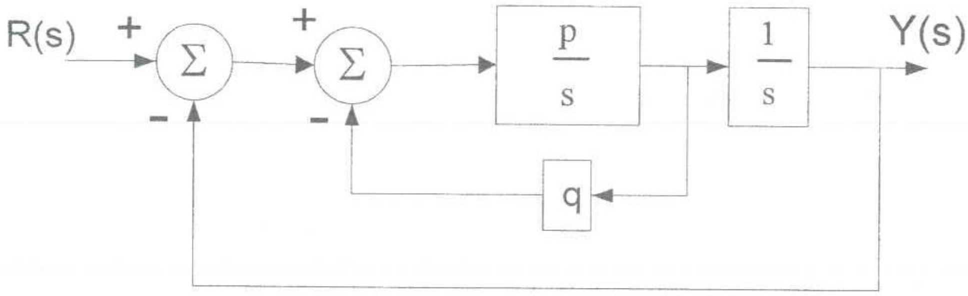

Figure Q1(b)

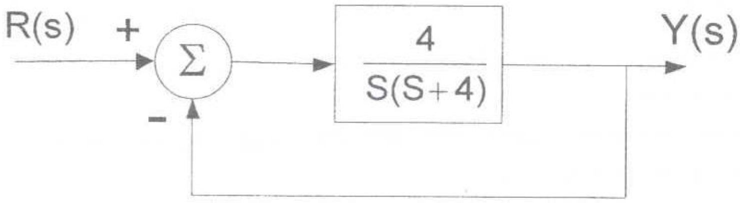

Figure Q1(c1)

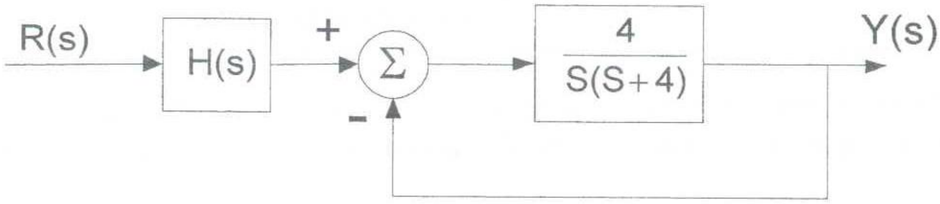

Figure Q1(c2)

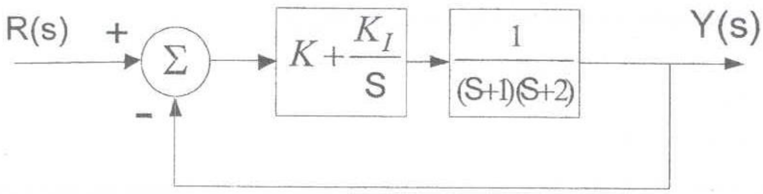

Figure Q2(b)

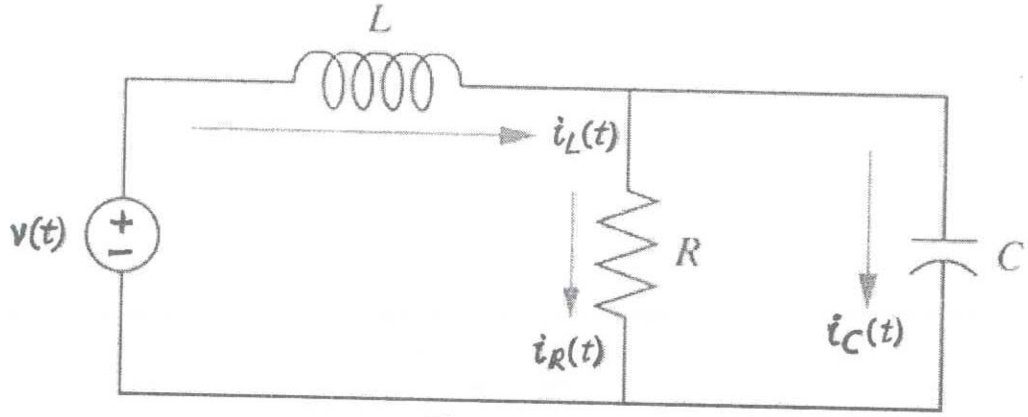

Figure Q2(c)

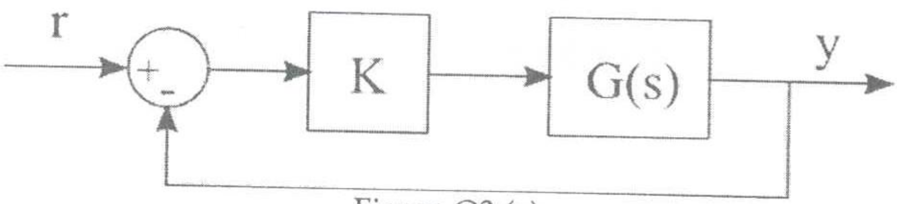

Figure Q3 (a)

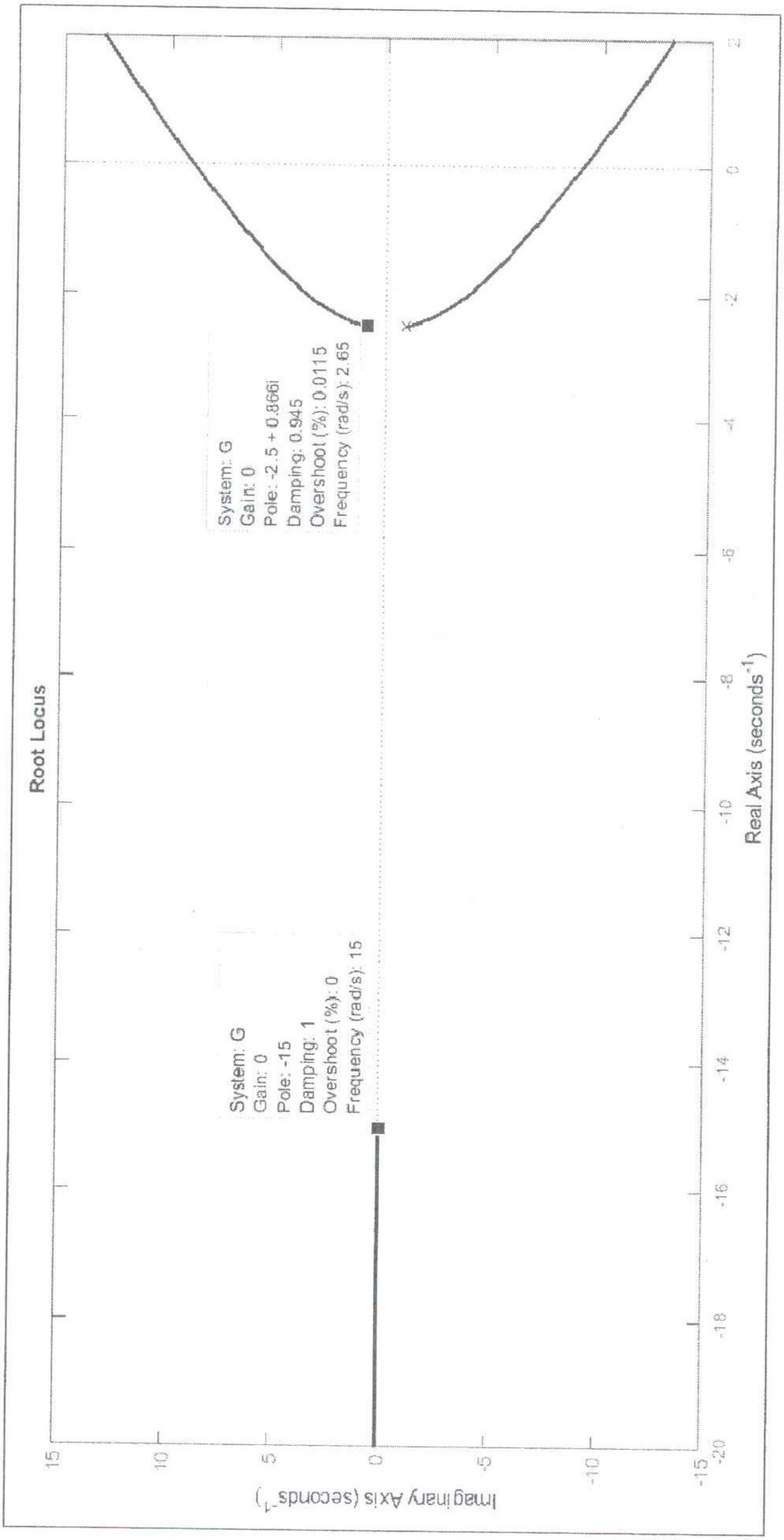
Root Locus
System: G
Gain: 0
Pole: -2.5 + 0.866i
Damping: 0.945
Overshoot (%): 0.0115
Frequency (rad/s): 2.65
System: G
Gain: 0
Pole: -15
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 15
Real Axis (seconds^-1)
Imaginary Axis (seconds^-1)

Figure Q3.(b)

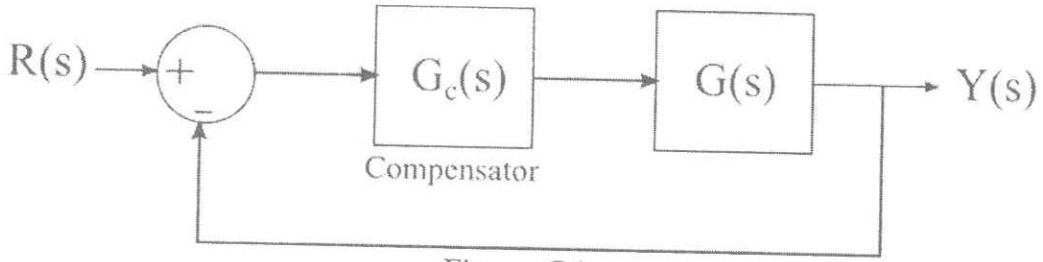

Figure Q4.

Formulas you may require:
(All notations have their usual meaning)

$$M_P = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$t_P = \frac{\pi}{\omega_d}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\phi_{PM} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}$$

For the lead compensator

$$G_{lead}(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

$$\phi_{lead,max} = \sin^{-1}\frac{1-\beta}{1+\beta}$$

$$|G_{lead}(j\omega_{max})| = \frac{1}{\sqrt{\beta}}$$

$$\omega_{max} = \frac{1}{T\sqrt{\beta}}$$

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-at}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$