UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: December 2022

Module Number: IS6303 **Module Name: Mathematical Modelling**

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1.

a) i.) Briefly explain the stages of 'Mathematical Modelling Process'.

ii.) Name two problems that might be modelled mathematically. Why do you think mathematics may provide a key to solving each problem? What is the added value in each case?

[3 Marks]

b) A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 2 hours, the number of bacteria increased to 1000.

i.) What is the number of bacteria in 5 hours?
ii.) How long will it take to reach 25,000 bacteria?

[3 Marks]

c) i.) Clearly state the 'Buckingham Pi Theorem'.

ii.) The terminal velocity (V) of a raindrop falling from a motionless cloud is believed to depend on the air viscosity (μ), size of the raindrop (r ; assuming a sphere shape), gravitational acceleration (g) and the density (ρ). Using the Buckingham Pi theorem, find a relation between the terminal velocity of the raindrop and the other parameter(s) of the system.

[6 Marks]

Q2.

a) i.) State the advantages and disadvantages (2 each) of 'Linear Programming' used in mathematical modelling.

ii.) Briefly explain the steps of solving a linear programming problem using 'Graphical Method'.

[3 Marks]

b) A patient in a hospital is required to have at least 90 units of drug A and at least 120 units of drug B each day. Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. Now suppose that both M and N contain an undesirable drug C (which needs to be minimized), 3 units per gram in M and 1 unit per gram in N.

i.) State the objective function and the problem constraints.

ii.) Using the graphical method, find the amount of substances M and N (in grams) that should be mixed to meet the stated daily requirements whilst minimizing the intake of drug C. How many units of the undesirable drug C will be in this mixture?

iii.) Suppose that overdoses are potentially harmful and the patient is to receive between 90 and 100 units of drug A and between 120 and 130 units of drug B. How does this change the feasible region?

[9 Marks]

Q3.

a) Briefly explain about 'Balanced and Unbalanced Transportation Problems' and the use of 'Dummy demand point and Dummy supply point' in transportation problems.

[2 Marks]

b) Briefly explain the procedures of 'Vogel's Approximation Model (*VAM*)' and 'Stepping-Stone Solution Method' used to solve transportation problems.

[2 Marks]

c) Ceylon Power Company has three electric power plants that supply the needs of four cities. Electricity supply of power plants 1, 2 and 3 are 30 million, 50 million and 40 million (in kWh) respectively. The peak power demands (in kWh) of cities 1, 2, 3 and 4 are 50 million, 20 million, 30 million and 30 million respectively. The costs of sending 1 million kWh of electricity from a plant to a city in 10^3 Rupees is given in the table below. To minimize the cost of meeting each city's peak power demand, formulate a balanced transportation problem in a transportation tableau and represent the problem as a linear programming model.

From	To			
	City 1	City 2	City 3	City 4
Plant 1	8	11	11	9
Plant 2	9	11	13	7
Plant 3	14	9	11	5

i.) Set up the transportation tableau for this problem and determine the initial solution using the Vogel's Approximation Model.

ii.) Find the total cost for the initial feasible solution.

iii.) Solve the problem using the Stepping-Stone Solution Method.

iv.) Are there multiple optimal solutions? If so, explain and identify them.

[8 Marks]

Q4.

a) Head of a department has 5 instructors to be assigned to 4 different courses. All of the instructors have taught the courses in the past and have been evaluated by the students. The rating for each instructor for each course is given in the following table (a perfect score is 100). The department head wants to know the optimal assignment of instructors to courses that will maximize the overall average evaluation. The instructor who is not assigned to teach a course will be assigned to grade exams. Solve this problem using the assignment method. Who should be assigned to grade exams?

Instructor	Course			
	A	B	C	D
1	80	75	90	85
2	95	90	90	95
3	85	88	95	91
4	93	92	80	84
5	91	91	93	88

[7 Marks]

c) i.) Briefly explain what is meant by 'a difference equation'.

ii.) Briefly explain how can you identify the linear stability of a fixed point.

iii.) Find the equilibrium point, if it exists, for the following dynamical system. State and prove whether the equilibrium point is stable or unstable.

$$u(n) = 0.9\,u(n-1) - 3.5$$

[5 Marks]

Q5.

a) A building project is modelled by the activity network shown in the Figure Q5 below. The activities involved in the project are represented by the arcs. The numbers in brackets on each arc gives the time, in days, taken to complete the activity.

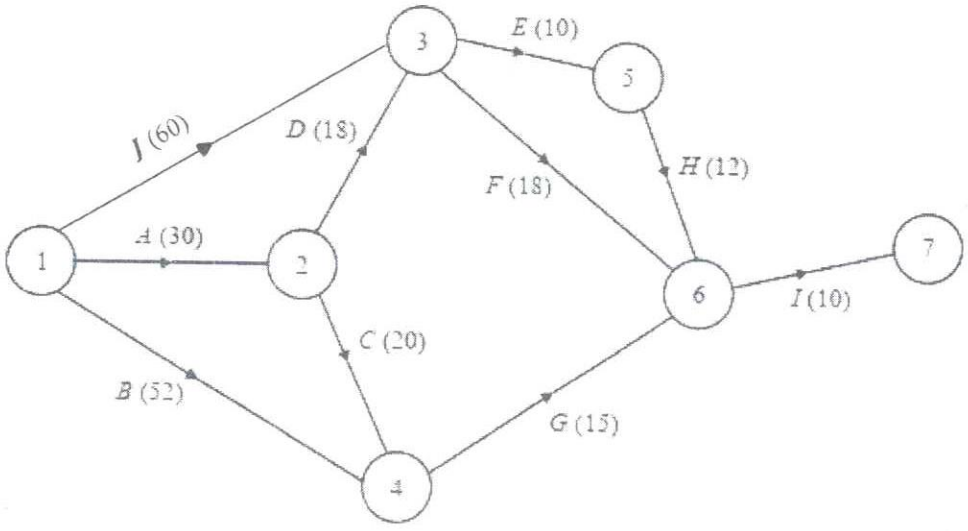

Figure Q5

i.) Construct the primal linear programming problem in order to find the shortest path of the network diagram.

ii.) Find the shortest path of the network diagram using the dual problem of the primal problem obtained in part (i). Hence, find the minimum time taken to complete the project in days (feasible solution).

[7 Marks]

b) The Table below shows the distances, in meters between six nodes A, B, C, D, E and F of a network.

	A	*B*	*C*	*D*	*E*	*F*
A	-	10	12	13	20	9
B	10	-	7	15	11	7
C	12	7	-	11	18	3
D	13	15	11	-	27	8
E	20	11	18	27	-	18
F	9	7	3	8	18	-

i.) Construct the network diagram according to the distance matrix.

ii.) Use Prim's algorithm, starting at A to solve the minimum connector problem for this Table of distances. Explain your method and indicate the order in which you selected the edge.

iii.) Determine your minimum spanning tree and find its total length.

iv.) State whether your minimum spanning tree is unique. Justify your answer.

[5 Marks]