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University of Ruhuna- Faculty of Technology

Bachelor of Information & Communication Technology

Level 1 (Semester 2) Examination, December 2023

Academic year 2021/2022

Course Unit: TMS 1233 - Discrete Mathematics **Duration: 03 hours**

Answer all questions.

Q1)

1.1) Define the term 'Propositions' and state five logical operators with their symbols used to combine propositions. (16 marks)

1.2) Let *p, q, r, s* and *t* be the propositions defined as follows,

- ***p*** : You get more than 50% out of 40 marks for continuous assessments.
- ***q*** : You have 80% attendance to the lectures for mathematics module.
- ***s*** : You get the approval submitted medical to student request committee for three lectures which are absent.
- ***r*** : You get eligibility to sit for the final exam of mathematics module.
- ***t*** : You get pass grade for the mathematics module.

Write the following propositions using *p, q, r, s* & *t* and logical connectives.

a) You get eligibility to sit for the final exam of mathematics module, when you get more than 50% out of 40 marks for continuous assessments and 80% attendance to the lectures for mathematics module .

b) You do not get eligibility to sit for the final exam of mathematics module, only if you get more than 50% out of 40 marks for continuous assessments and do not get approval submitted medical to student request committee for three lectures which are absent.

c) To get pass grade for the mathematics module, it is necessary for you to more than 50% out of 40 marks for continuous assessments.

d) You should have either 80% attendance to the lectures for mathematics module or the approval submitted medical to student request committee for three lectures which are absent.

e) Getting less than 50% out of 40 marks for continuous assessments or do not getting approval submitted medical to student request committee for three lectures which are absent is sufficient for do not getting eligibility to sit for the final exam of mathematics module.

f) You will get pass grade for the mathematics module if and only if you either get eligibility to sit for the final exam of mathematics module and you get more than 50% out of 40 marks for continuous assessments.

(48 marks)

1.3) State the **converse**, **contrapositive**, and **inverse** of each of the following conditional statements.

Statement A: "I come to lecture whenever there is lecture boycott by students."

Statement B: "For $x = 2$, it is sufficient for $x^2 = 4$." (24 marks)

1.4) Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 1101 1001 0100 and 0111 0011 1101.

(12 marks)

Q2)

2.1) Define the terms of Tautologies, Contradictions and Contingencies in Propositional Equivalences (15 marks)

2.2) Show that $\boldsymbol{p \leftrightarrow q}$ and $(\boldsymbol{p} \rightarrow \boldsymbol{q}) \wedge (\boldsymbol{q} \rightarrow \boldsymbol{p})$ are logically equivalent using truth table.

(15 marks)

2.3) Show that $\neg(\boldsymbol{p} \vee (\neg \boldsymbol{p} \wedge \boldsymbol{q}))$ and $\neg \boldsymbol{p} \wedge \neg \boldsymbol{q}$ are logically equivalent by developing a series of logical equivalences. (Note: Do not use a truth table to establish this equivalence).

(35 marks)

2.4) a) Express the two De Morgan's Laws in logical equivalences. (10 marks)

b) Determine the satisfiability of the following compound propositions and justify your answer. (Hint: you can use truth tables) (25 marks)

i. $(\boldsymbol{p} \vee \neg \boldsymbol{q}) \wedge (\boldsymbol{q} \vee \neg \boldsymbol{r}) \wedge (\boldsymbol{r} \vee \neg \boldsymbol{p})$

ii. $(\boldsymbol{p} \vee \boldsymbol{q} \vee \boldsymbol{r}) \wedge (\neg \boldsymbol{p} \vee \neg \boldsymbol{q} \vee \neg \boldsymbol{r})$

Q3)

3.1) What is the truth value of $\exists x P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4? Explain your Answer.

(12 marks)

3.2) Determine the truth value (True/False) of each of these statements if the domain consists of all real numbers. Provide explanations using examples.

(28 marks)

a) $\exists x\ (x^3 = -8)$

b) $\exists x\ (x^4 < x^2)$

c) $\forall x\ ((-x)^2 = x^2)$

d) $\forall x\ (3x > x)$

3.3) Let $S(x)$ be the statement "x is a student in this class", let $D(x)$ be the statement "x has studied disrete mathematics", and let $J(x)$ be the statement "x has taken a course in Java". Express each of the following sentences in terms of $S(x)$, $D(x)$, and $J(x)$ quantifiers, and logical connectives. The domain for quantifiers consists of all the people.

(35 marks)

a) Some student in this class has taken a course in Java.
b) Every student in this class has taken a course in Java.
c) Every student in this class has taken either discrete mathematics or Java.
d) There is a student in this class who has not taken Java and taken discrete mathematics.
e) There is a student does not study Java or discrete mathematics.

3.4) Let $B(x)$ be the statement "x is a boy" and let $G(x)$ be the statement "x is a girl", where the domain consists of the students in your university. Express each of these quantifications in English. (25 marks)

a) $\exists x B(x)$

b) $\forall x B(x)$

c) $\neg \exists x B(x)$

d) $\exists x \neg G(x)$

e) $\neg \forall x B(x)$

Q4)

4.1) Translate the equality, $1 \cdot 0 + \overline{(0 + 1)} = 0$, into a logical equivalence. (10 marks)

4.2) Prove the following identities using the other identities of Boolean algebra.

a) **Absorption law;** $x(x + y) = x$

b) **Distributive law**; $x + yz = (x+y)(x+z)$ (40 marks)

4.3) Find the sum-of-products expansion for the following functions using truth table.

(30 marks)

a) $F(x, y, z) = \overline{(x + y)} . z$

b) $F(x, y, z) = (x + y + z).(\bar{x} + \bar{y}\,\bar{z})$

4.4) Construct circuits from NOT gates, AND gates, and OR gates to produce the following outputs.

(20 marks)

a) $x\,y + \bar{x}\bar{y}$

b) $\bar{x}.\overline{(y + \bar{z})}$

c) $(x + y + z).(\bar{x}\bar{y}\,\bar{z})$

d) $y.(x + x\bar{y} + \bar{z})$

Q5)

5.1) Explain the rule of inference called "Modus tollens". (10 marks)

5.2) State which argument form (rule of inference) is the basis of the following argument and use variables to represent it. (50 marks)

a) Whales lives in sea and whales are mammals. Therefore, whales are mammals.

b) It is either outside temperature less than 40 degrees today or the depletion of ozone layer is higher than last year. It is higher than 40 degrees outside temperature today. Therefore, the depletion of ozone layer is higher than last year.

c) Glenn Mawell is an excellent cricketer. If Glenn Mawell is an excellent cricketer, then he can be a man of match. Therefore, Glenn Mawell can be a man of match.

d) Jamila will work at a IT company next year. Therefore, next year Jamila will work at a IT company or she will be a Demonstrator.

e) If the student cover the lecture material according to lecturer guidance, then student can understand discrete mathematics. If he can understand discrete mathematics, He will get A+ grade for discrete mathematics. Therefore, if the student cover the lecture material according to lecturer guidance, then He will get A+ grade for discrete mathematics.

5.3) Show that the premises "You involve exam offense activity at the exam and you will not get the academic suspend for few years," If invigilator catch you during those activities, then you will get the academic suspend for few years," and "If invigilator could not catch you during those activities, then you will not pass the exam with good grade" lead to the conclusion ", You will not get pass the exam with good grade." (20 marks)

5.4) Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \land S(x)))$ and $\forall x(P(x) \land R(x))$ are true, then $\forall x(R(x) \land S(x))$ is true. (20 marks)

Q6)

6.1) Find the cardinal number of the following sets. (15 marks)

a) $A = \{x : x \in Z, -5 < x \leq 10\}$

b) $B = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

c) $C = \{x : x \in Z^+, x \leq 70\}$

6.2) Write down the power set of the following sets. (15 marks)

a) $A = \{0, 1, 2\}$

b) $B = \{\emptyset\}$

c) $C = \{\emptyset, \{\emptyset\}\}$

6.3) State whether each of these statements is true or false. (35 marks)

a) $0 \in \emptyset$

b) $\emptyset \in \{0\}$

c) $\{0\} \subset \emptyset$

d) $\emptyset \subset \{0\}$

e) $\{0\} \in \{0\}$

f) $\{0\} \subset \{0\}$

g) $\{\emptyset\} \subseteq \{\emptyset\}$

6.4) a) Construct a membership table to show that the Absorption law holds. (15 marks)

$$A \cup (A \cap B) = A \ , \ A \cap (A \cup B) = A .$$

b) Use set builder notation and logical equivalences to establish the first De Morgan law,

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B} .$$

(20 marks)

**** **End of the Examination Paper******