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University of Ruhuna-Faculty of Technology

Bachelor of Engineering Technology (Honors) Degree

Level 1 (Semester II) Examination, December 2023

Academic Year 2021/2022

Course Unit: TMS1213 – Applied Calculus II **Duration: 3 hours**

- This paper contains **six (06)** questions.
- Answer **all** the questions.
- Calculators are **not allowed.**

Q1.

Let $f(x) = x(x-1)(x-2)$

a) What are the x intercepts and y intercepts of the function $f(x)$. (20marks)

b) Find all the critical points of the function $f(x)$. ($\sqrt{3} \approx 1.7$) (70marks)

c) Based on the information you have obtained from above parts, sketch the shape of the function $f(x)$ on a 2D cartesian plane. (10marks)

Q2.

Integrate the following given integrations to obtain the given answers.

a)

$$\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx = \frac{1}{4}(\pi - \ln 4)$$

(50marks)

b)

$$\int_0^1 \frac{1}{(4-x^2)^{\frac{3}{2}}} \, dx = \frac{\sqrt{3}}{12}$$

; Use $x = 2\sin\theta$ (50marks)

Q3.

a) Evaluate the arc lengths of the following curve over the given interval.

$f(x) = \frac{2}{3}(x^2+1)^{\frac{3}{2}}$ over the interval $1 \leq x \leq 4$

(50marks)

b) Evaluate the area of the surface of revolution formed by revolving the following curve about the x axis over the given interval.

$f(x) = \sqrt{x}$ over the interval $0 \leq x \leq 4$

(50marks)

Q4.

The curve $y = x^2 - 2x$ intersects the line $y = x$ at origin and at the point B. Also, the curve crosses the x-axis at the origin, and at the point A.

a) Find the coordinates of the points A and B. (20marks)

b) Sketch the curve and the line on same graph. (30marks)

c) Find the area of the region enclosed by the curve and the line $y = x$ is $\frac{9}{2}$. (50marks)

Q5.

Consider the region in the 1st quadrant of the 2D Cartesian coordinates plane, bounded by $y = x^2 - 4$ and $x = 1, y = 1$ and $y = 5$. Calculate the volume of the solid formed by revolving the bounded region about y - axis. (100marks)

Q6.

a) Water is being heated in a kettle. At time t, the temperature of the water is θ°C. The rate of increase of the temperature at any time t is modelled by the differential equation,

$$\frac{d\theta}{dt} = -A\theta$$

where A is a positive constant.

Given that $\theta = 20$°C when $t = 0$, solve the above differential equation to show that,

$$\theta = 19 + e^{-At}$$

(25marks)

b) Consider the second order differential equation,

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$$

i) Find the values of k such that $y = e^{kx}$ is a solution of the given differential equation. (40marks)

ii) Hence find the general solution. (05marks)

iii) Determine the values of the constants of the general solutions using the initial conditions,

$$Y(0) = 2 \text{ and } y'(0) = 1.$$

(30marks)

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