UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 8 Examination in Engineering: July 2022

Module Number: EE8212 **Module Name: Optimization Techniques for Engineers**

[Three Hours]

[Answer **all questions**, each question carries **10 marks**]

Q1 a) State two (2) characteristics of a two person zero sum game.

[2.0 Marks]

b) Due to the shortage of fuel stocks in the country, each filling station has a limitation of providing fuel for each vehicle. Two filling stations at Kalegana and Karapitiya, have maximum limitations of 10 L and 12 L per car, respectively, when issuing fuel for them. These two filling stations refuel vehicles based on 'first come, first served' basis. Sunil and Kumara decide to refuel their cars. Sunil is closer to both filling stations. Here are the limitations of the situation.

- If both go to the same filling station, the first one will get the maximum amount of fuel, while the other person gets nothing.
- If they choose two different filling stations, both can refuel their vehicles with the maximum limit of each of the filling station.

Note: Assume both Sunil and Kumara leave their homes at the same time and the traveling speed is the same.

Model the above situation as a game and prepare a payoff matrix. Determine the optimal strategy for each player. (Answers without steps/ explanations carry no marks)

[4.0 Marks]

c) Consider the following two-player game with the payoff matrix for player 1 (row player) given in Table Q1-c.

Table Q1-c

	S1	S2	S3	S4	S5
S1	5	9	8	12	6
S2	6	8	10	5	4
S3	0	7	9	12	6
S4	10	8	12	11	5
S5	11	7	10	9	5

Determine the optimal strategy for player 2.

[4.0 Marks]

Q2 A company produces two types of kitchen utensils in three different plants. Plants A, B and C are used for heating, finishing, and cutting, respectively, to produce the two types of products, which are spoons (product 1) and measuring cups (product 2). Product 1 requires some production capacity in plants A, B and C, while Product 2 requires some production capacity in plants A and B. The number of hours in each day available for the two products in plants A, B and C are 20, 18 and 8 hours, respectively. Product 1 requires 2 hours of plant A, 1 hour of plant B and 1 hour of plant C per batch produced. Product 2 requires 1 hour of plant A and 1 hour of plant B per batch produced. Profit per batch of product 1 is Rs 700,000 and that of product 2 is Rs 400,000.

a) Formulate this problem as a linear programming problem.

[2.0 Marks]

b) Using the simplex tabular method, determine how many spoons and measuring cups are required to produce to maximize the profit.

[8.0 Marks]

Q3 a) Consider the following Knapsack problem with notations of,

w_i - weight of each type-i item, i = 1, 2, . . ., N
γ_i - value associated with each type-i item, i = 1, 2, . . ., N
C - weight capacity of the knapsack.

i) Formulate the knapsack problem using the above notations.

[2.0 Marks]

ii) A knapsack which has a capacity of 7 kg can be loaded with one or more of three items with weights w_i = [1, 2 ,5] and their associated values of γ_i = [10, 21,51], respectively (where i = 1, 2, 3). The unit weight w_i is in kilograms and unit value γ_i is in rupees for each item. How should the knapsack be loaded in order to maximize the value? (Answers without mathematical formulations and calculations carry no marks)

[5.0 Marks]

b) A travel path for a salesperson is shown in Figure Q3-b, where location O is the starting city and location T is the end city. The numbers give the profit of following each path in thousand rupees. Using dynamic programing techniques, find the path/paths the salesperson should follow from starting point O to end point T in order to maximize the profit. (Answers without mathematical formulations and calculations carry no marks)

[3.0 Marks]

Q4 a) Briefly explain the following under graph theory terminologies with an example figure.

i) Weakly Connected Graph
ii) Strongly Connected Graph

[2.0 Marks]

b) A flight from Heathrow Airport to Shanghai Airport can follow different paths as shown in the following table. Wi represents the intermediate waypoints located in the airspace. (waypoint represents an airspace location which is used to define

the air route/path of an aircraft for navigation purposes). W1 represents Heathrow Airport and W8 represents Shanghai Airport. The time to travel (in hours) for a direct flight from Wi to Wj is given by $(i,j)^{th}$ entry in Table Q4-b (note that there is no direct path between some nodes). Find the fastest routes from W1 to all other waypoints. What is the fastest route from Heathrow Airport to Shanghai Airport?

Directions: You are required to produce a graph and use Dijkstra's algorithm to solve this problem. Clearly show the update and visit process for each step in separate graphs.

Table Q4-b

Waypoint	W1	W2	W3	W4	W5	W6	W7	W8
W1		5	6	3				
W2	5		2		4	5		
W3	6	2		2	1		2	
W4	3		2			4	5	
W5		4	1			3		3
W6		5		4	3		1	
W7			2	5		1		1
W8					3		1	

[4.5 Marks]

c) An oil pipe network in a central city along with the pipe capacities in liters is shown in Figure Q4-c.

i) Using Ford-Fulkerson algorithm, find the maximum amount of liters of oil that can be pumped from node A to node F.

ii) Use Max-Flow-Min-Cut theorem to prove that you have arrived to the optimum solution.

[3.5 Marks]

Q5 a) State two(2) characteristics of an assignment problem.

[2.0 Marks]

b) 'Red cabs' taxi firm has five taxis available, and five customers are requesting to be picked up as soon as possible. One taxi can only pick up one customer. Estimated time each taxi takes to pick up customers is shown in Table Q5-b. How should these taxis be assigned to the customers in order to minimize the total waiting time?

Table Q5-b

	Taxi 1	Taxi 2	Taxi 3	Taxi 4	Taxi 5
Kamal	10	22	12	18	19
Namal	15	18	21	15	8
Sahan	21	19	23	11	12
Gayan	12	20	21	17	18
Aruna	20	17	12	15	11

[5.0 Marks]

c) Repeat the assignment process if Taxi 2 is not available.

[3.0 Marks]

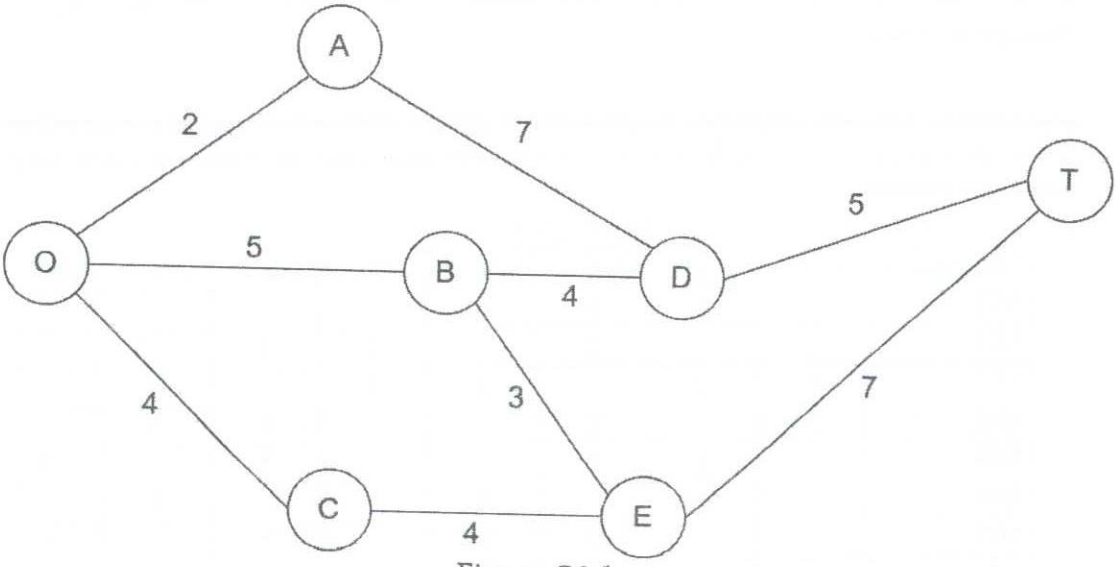

Figure Q3-b

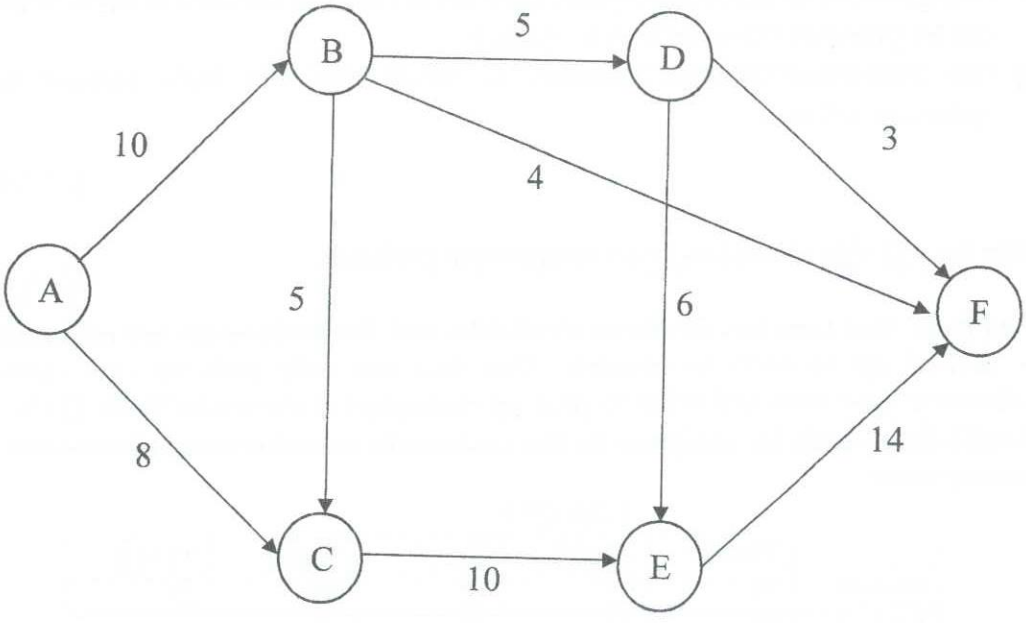

Figure Q4-c