UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 8 Examination in Engineering: September 2023

Module Number: EE8212 **Module Name: Optimization Techniques for Engineers**

[Three Hours]

[Answer **all questions**, each question carries **10 marks**]

Q1 a) State two (2) characteristics of a game.

[2.0 Marks]

b) Sunil and Kamal, two neighboring farmers engage in the cultivation of various crops while competing in the same market. They are tasked with deciding which crops to plant among three options: Grain, Corn, and Wheat, and each of them has three potential strategies for crop selection. Sunil typically specializes in planting Corn and Wheat, yielding average monthly profits of 8,000 rupees and 7,000 rupees, respectively. In contrast, Kamal generally opts for Grain and Corn, which generates average monthly profits of 9,000 rupees and 10,000 rupees, respectively. Due to market demand, Sunil decides to introduce a new crop: Grain, while Kamal chooses to cultivate a new crop: Wheat. After thorough market analysis, Sunil anticipates an average monthly profit of 9,000 rupees from Grain, while Kamal expects to earn an average monthly profit of 11,000 rupees from Wheat from their lands.

Additionally, they have determined that if both cultivate the same crops, they will each achieve similar average monthly profits: 5,000 rupees for Grain, 8,000 rupees for Corn, and 7,000 rupees for Wheat. Furthermore, they have found that if both choose to grow the new crop, they can each earn an average monthly profit of 9,000 rupees.

Their income depends on the chosen crop planting strategy as described above. Both Sunil and Kamal are focused on optimizing their profits through strategic crop selection. Model the above situation as a game and prepare a payoff matrix. Determine the optimal strategy for each player by eliminating dominated strategies. (Answers without steps/explanations carry no marks)

[4.0 Marks]

c) Consider the two-player game with the payoff matrix for player 1 (row player) given in Table Q1-c. Use the graphical method to identify the optimal strategies for both players and determine which player has the advantage of the game.

[4.0 Marks]

Q2 A bakery called 'sweet treats' is famous for two delightful bakery items: cakes, and cookies. However, it was facing a challenge. Despite the high demand for their delightful treats, they had to optimize their production process to ensure that they didn't run out of ingredients and maintain profitability. The owner Mrs. Sharma decided to maximize her bakery's profit while considering the remaining stocks of ingredients as flour, sugar, and eggs each in limited supply. The total flour, sugar and eggs required for cakes, and cookies must not exceed the available stocks of 18 kg, 42 kg and 24 kg for the month of October.

To produce 1 kg of cake, the requirements are 2 kg of flour, 2 kg of sugar, and 3 kg of eggs. For a batch of cookies, the requirements are 1 kg of flour, 3 kg of sugar, and 1 kg of eggs. The selling price is 300 rupees for 1 kg of cake and 200 rupees for a batch of cookies.

a) Formulate this problem as a linear programming problem.

[2.0 marks]

b) Use the algebraic steps of the simplex method to systematically solve the above problem and determine the amount of cakes, and cookies which can produce to maximize the profit.

[5.0 marks]

c) To confirm the values obtained in part b), use the simplex tabular approach.

[3.0 marks]

Q3 a) Give two (2) examples of the Knapsack problem applications in brief.

[2.0 Marks]

b) Sarah, a student from university of Agel wants to decide on her module selection for the next semester based on her current results. From all other modules (excluding only the management modules) she has earned 3.25 GPA with a total credit of 120. She has covered all management modules and wanted to know the remaining credit balance for the next semester for other elective modules to reach the minimum credit requirement of 130 to complete the degree. Management module code, number of credits and the grade point earned by Sarah are listed in Table Q3-b.

As mentioned in the student handbook, Sarah found that management modules can only cover up to a maximum of 6 credits for the GPA calculation. However, her priority is to maximize her GPA.

(i) Which of the modules listed in Table Q3-b should be chosen to maximize her GPA? (Answers without mathematical formulations and calculations carry no marks)

[4.5 Marks]

(ii) How many credits does she want to complete during the next semester to reach the minimum credit requirement to complete the degree?

[0.5 Marks]

c) Amal works as a courier and needs to deliver a package from node-S to node E. There are multiple paths connecting these nodes as shown in Figure Q3-c, each with a time and cost associated with it. The values are represented on the edges of the network (first value - time, second value - cost). Amal wants to minimize both the time and cost of the delivery. Using dynamic programing techniques, find the path/paths Amal should follow from starting node S to end node E. (Answers without mathematical formulations and calculations carry no marks)

[3.0 Marks]

Q4 a) Give a brief explanation of following using an example figure under 'graph theory terminology'.

(i) Weakly Connected Graph

(ii) Strongly Connected Graph

[2.0 Marks]

b) Imagine you are the air traffic controller at a major international airport. An aircraft has encountered a critical technical issue during mid-flight and must make an emergency landing at the nearest airport to ensure the safety of all passengers and crew members. Ni represents the airports within the aircraft's range. Wi represents the intermediate waypoints located in the airspace, which are used for navigation purposes. The path between two nodes (airports/waypoints) is called the air route, and the time to travel (in hours) in an air route is given in the entries in Table Q4-b.

As the air traffic controller of this region, your task is to find the nearest suitable airport for the emergency landing for a flight at waypoint W1 using Dijkstra's algorithm. (Clearly show the update and visit process for each step in separate graphs.)

(i) What airport is the emergency flight's closest point of landing?

[4.5 Marks]

(ii) What is the shortest route and how long does it take to reach the landing airport?

[0.5 Marks]

c) As an electrical engineer, you are asked to design the wiring network for a computer system. The goal is to minimize the total length of the wire required to connect various components within the system while ensuring that all components are interconnected without forming loops or cycles. The wire lengths (in meters) between components are illustrated in Figure Q4-c.

Find the shortest amount of wire that will accomplish the aforementioned goal, and graph the edges of the connections.

[3.0 Marks]

Q5 a) Consider the following Assignment problem with notations of,

$X_{ij} = \begin{cases} 1 \; ; if \; job \; j \; is \; performed \; by \; the \; person \; i \\ 0 \; ; Otherwise \end{cases}$

C_{ij} - Cost of performing jth job by ith person

Where, i, j = 1,2,3,....., n

Use the aforementioned notations to formulate the assignment problem.

[2.0 Marks]

b) A hospital has five dedicated healthcare professionals (Nurses A, B, C, D, and E) and five essential tasks that need to be assigned for a shift. Each nurse has a different skill set and experience level, thus the time required (in minutes) to complete each task is different and the values are shown in Table Q5-b. How should these nurses be assigned to the tasks in order to minimize the total time?

[5.0 Marks]

c) Assume that the values given in Table Q5-b are the skill level of each nurse. Explain how you can assign tasks to nurses to maximize the total skill level of the assigned persons. (You only need to explain the steps and a final answer is not required)

[3.0 Marks]

Table Q1-c

	S1	S2	S3	S4
S1	4	3	2	3
S2	1	1	5	6
S3	5	2	4	4
S4	3	1	2	3

Table Q3-b

Module	Number of credits	Grade point
MG 8241	2	2.3
MG 6305	3	2
MG 4201	2	3.3
MG 8132	1	3.7
MG 5244	2	4

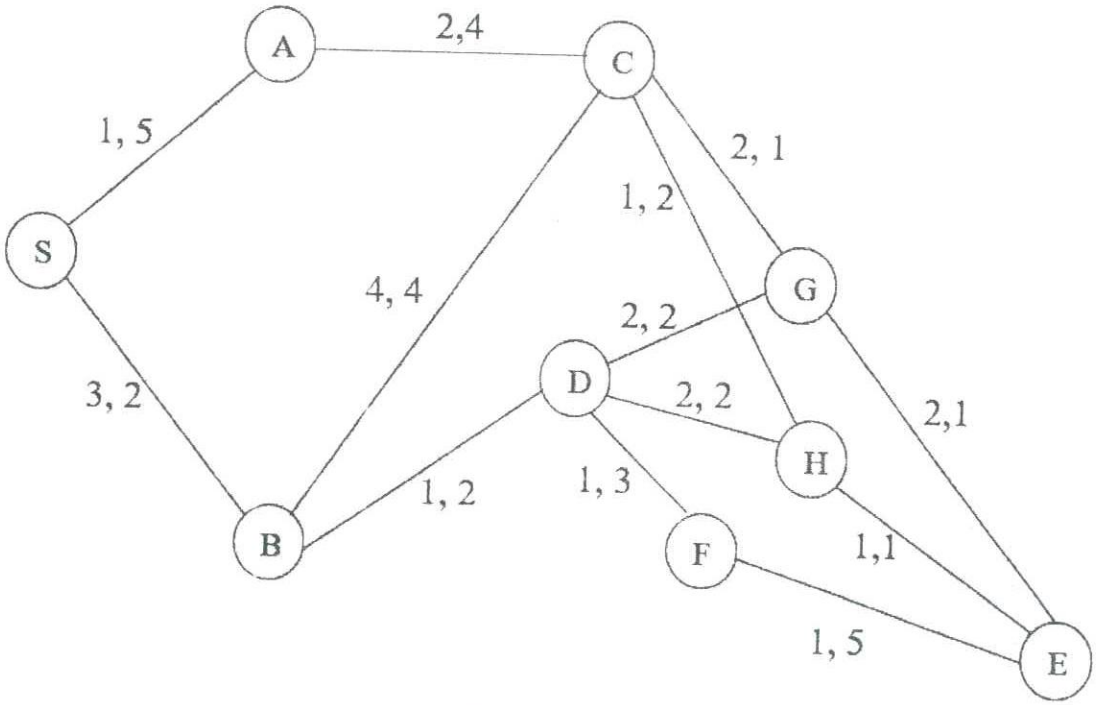

Figure Q3-c

Table Q4-b

Waypoint /Airport	W1	W2	W3	W4	W5	N1	N2	N3
W1		7	5	3				
W2	7		2		2	1		
W3	5	2		1	2	5	4	
W4	3		1			6	7	
W5		2	2			2		3
N1		1	5	6	2		2	3
N2			4	7		2		2
N3					3	3	2	

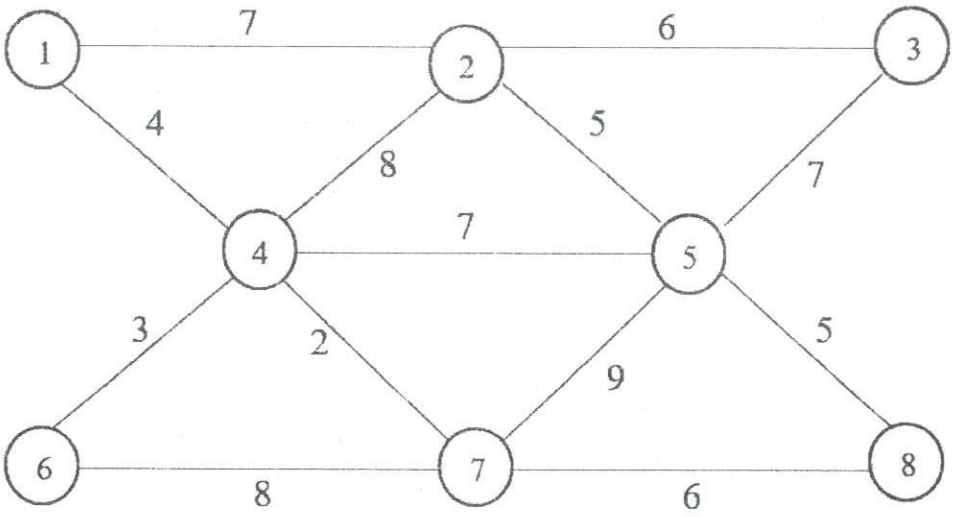

Figure Q4-c

Table Q5-b

Nurse\ Task	Task 1	Task 2	Task 3	Task 4	Task 5
Nurse A	8	20	10	16	17
Nurse B	13	16	19	13	6
Nurse C	19	17	21	9	10
Nurse D	10	18	19	15	16
Nurse E	18	15	10	13	9